Disclosure of Hard vs. Soft Information*

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Abstract

This paper studies voluntary disclosure when (a) there is uncertainty about managerial propensity to report truthfully, (b) some components of the firm's value may be certified for a cost ("hard"), (c) other components may be disclosed but not certified free of misstatements ("soft"). We establish that untruthful managers are more likely to certify hard information and that, among truthful managers, those with more favorable soft information also certify more. Even if certification is costless, unraveling to a complete certification of the hard information may not occur. We develop several testable predictions linking the presence or absence of a certification to managerial credibility, earnings quality, the magnitude or likelihood of frauds and market reactions to disclosures. The model has many natural applications, including credit ratings, press releases vs. financial statements, auditing choice, going dark and voluntary asset appraisals.

1 Introduction

Most practical instances of strategic communication involve the following ingredients. The information is multidimensional; some pieces of information can be certified at a cost; some pieces of information cannot be certified and therefore an informative communication requires trust; there is uncertainty about the credibility of the sender, i.e., about the propensity that a sender who faces conflicting economic incentives tells the truth.

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The interaction between these two types of information presents several obvious questions of interest. Should the decision to certify hard information depend on any soft information, *even if* that soft information cannot be certified and does not affect the value of the component being certified? Should a sender with more discretion to manipulate soft information certify more, or less? And, if the answer to either of these questions is positive, what does certification indicate about the sender's credibility, and about the component of the information that cannot be certified?

We formally study strategic communication in this context. Since most market transactions can be described in these terms our analysis has numerous applications. Below we list several applications, beginning with our baseline example.

Publicly traded firms periodically release information to investors in the capital market. Some of that information is distributed in the form of a voluntary disclosure through formal channels, such as the firm's financial statements, all of which are subject to an independent audit and are filed with the Securities and Exchange Commission. Sometimes, the information is distributed through informal channels such as press releases, conference calls, conversations with analysts, investor meetings or even advertisements.

Another important choice is whether a firm should go public or remain private, or even delist its stock ("going dark"). While such a choice has many implications, an important one is that public firms should issue hard disclosures according to accepted accounting standards and the disclosure requirements of regulators or the exchanges where its stock is traded. A public firm that delists its stock from a stock exchange may, under certain circumstances, suspend its reporting obligations with the Securities and Exchange Commission.

Lastly, when a company issues debt, it may choose to certify the issuance by hiring a rating agency. The rating is however a very partial assessment of the issuance: the rating provides certification of the issuer's default probability but it does not provide certification on the issuer's profitability conditional on no default. The issuer has then two options: either to hire the rating agency and supplement the rating with some uncertified information about the quality of the firm's projects or not to hire the rating agency and provide only uncertified information to investors.

There are of course many other possible applications outside of the realm of financial accounting. We give here two additional examples. Marketing campaigns consists of two types of messages: (a) product characteristics that are verifiable and clearly specified in the warranty of the product or by the adoption of quality label and (b) more or less vague promises that customers cannot enforce either because the characteristics cannot be contractually specified or because customers can only gradually learn about them over time. Sellers will choose an appropriate level of product certification as a function of their credibility (or pre-existing brand image), the claimed quality of the product that they are selling, and the relative importance of soft characteristics.

Employees can provide to a new employer a number of verifiable signals about their qualifications such as a formal degree with a grade point average, or a standardized test score. Or, the worker may acquire such skills through self-study and experience. There are also personal skills that are difficult to certify, such as teamwork or leadership abilities, which are essential in the workplace. The theory explains why and when some workers would choose to certify some skills but also what the new employer should infer about the employee's non-certifiable skills or underlying reporting truthfulness.

1.1 Snapshot of the model and overview of main results

To fix ideas assume the sender is a firm's manager who has private information about both the firm's tangible assets and also the firm's customer satisfaction – which is presumably correlated with the firm's future revenues. The manager wishes to sell her firm for an exogenous reason and obtain the highest price. However, given the obvious conflict of interests, the manager's credibility is imperfect: some managers may report truthfully while others may manipulate the information whenever possible to achieve the highest price (this difference across managers may either be due to intrinsic honesty or some characteristics of internal controls or incentive systems.)

To overcome credibility problems, the manager can hire a reputable auditor. The auditor will verify the existence of certain tangible assets and the valuation methods that have been used. However, the auditor cannot certify customer satisfaction (or, for that matter, many of the firm's intangible assets.) The assets that cannot be certified are thus disclosed outside of the accounting system (e.g., milestones for ongoing research and development, replacement costs or executory contracts.) The firm chooses whether to hire this auditor to certify part of its business, and the market must decide whether to believe the firm's uncertified information.

The main results are stated and explained next. We show that untruthful managers certify *more*, relative to managers who must report truthfully. While this may seem counter-intuitive at first blush (since it is the manager with the most discretion who is willing to reduce that discretion the most), the intuition is tied to the substitution between formal certification and certain uncertified reports. A soft report indicating that "my total asset value is low" is always a credible signal that the manager is truthful and thus does not require any additional certification. A soft report indicating that "my total asset value is high" suffers from a comparably large price protection because it is likely to have been reported by an untruthful manager. Since certification removes part of this price protection (e.g., over those tangible assets certified by the auditor), it is more valuable to firms that would, absent certification, report higher total assets.

It follows that a truthful manager observing high asset values certifies more than truthful managers observing low asset values. The same intuition applies to untruthful managers: while they do not necessarily observe high total assets, they will report as if they did; hence, untruthful managers are the most willing to certify hard information. In doing so, they are also able to report more aggressively any remaining soft information that cannot be certified.

This has three important empirical implications which, to our knowledge, have not yet been tested. First, the decision to certify tangible assets should be correlated with the value of soft assets. Firms that certify their tangible assets should make more aggressive disclosures about other intangible assets such as customer satisfaction, research activities, etc. Conversely, the value of intangible assets must be higher among firms certifying tangible assets.

Second, since untruthful managers are more likely to certify, the credibility of the manager conditional on certification should decrease. This would seem to suggest that the market should discount more strongly intangible assets of certified firms. Yet, the opposite is true: claiming high intangible assets is, on average, less credible (and more strongly discounted) when the manager fails to certify tangible assets as opposed to when she certifies them. While a truthful manager is less likely to certify tangible assets, she is even less likely to both claim high intangible assets and not certify the tangible ones.

Third, we examine the magnitude of frauds, defined as the average overstatement by untruthful managers. Frauds are greater, though less frequent, when managers are more likely to be truthful. This has an important implication for policy-making since capital market regulations tend to follow the discovery of large-scale visible frauds, which in our model are indicative of more credible markets where direct regulation is likely to be the least necessary or desirable.

1.2 Detailed overview of the results

A more detailed overview of the results follows. In general, our model provides a general equilibrium analysis of communication where certification and misreporting are simultaneously chosen. These decisions are determined by three main factors: the initial credibility of the manager; the cost of certification and the volatility (or uncertainty) of both tangible and intangible assets.

A higher credibility naturally reduces the discount the market applies to uncertified assets, whether they are tangible or not. This in turn reduces the propensity to certify information but increases the magnitude of frauds in the uncertified market because untruthful managers exploit the greater credibility by both certifying less and by reporting higher values.

Higher certification costs naturally reduce the manager's tendency to certify. This reduction is particularly strong for untruthful managers who are more sensitive to certification costs because (unlike truthful managers) untruthful managers certify tangible assets even when intangibles are unfavorable. As a result, greater certification costs translate into a higher concentration of untruthful managers in the uncertified market, thus increasing the likelihood of misreporting in such market. Despite the greater misreporting risk, managers are able to induce higher prices in the uncertified market. The reason is that higher certification costs induce some certified firms to withdraw from the certified market thereby raising the average value of uncertified firms, as certified firms are more valuable than uncertified firms. In turn, the greater overall value of uncertified firms allows untruthful managers to engage in more aggressive reporting in the uncertified market. In summary, the increase in certification costs not only raises the chances of a fraud but also their magnitude, in the uncertified market.

We also find that the famous unraveling phenomenon described by both Sanford Grossman and Paul Milgrom in 1981 fails even when the cost of certification is zero. If markets do not perceive the decision to certify tangible assets as sufficiently good news on the firm's intangibles, the decision to remain uncertified always leads to higher prices than the certification option, even when the firm's tangible assets are underpriced in the uncertified market. Of course, this outcome is only possible if the manager's initial credibility is high enough to guarantee that mispricing in the uncertified market is not so acute.

A higher volatility of tangible assets reduces the price of uncertified firms because no certification is perceived by the market as bad news about the firm's tangible assets. To avoid this stronger price penalty, managers increase the propensity to certify tangible assets, particularly those who are untruthful. Hence, as the proportion of untruthful managers goes down in the uncertified market the chances of misreporting also diminish within uncertified firms. But not only the likelihood of misreporting goes down for these firms: since the market perceives uncertified firms more negatively when there is more volatility of tangible assets, untruthful managers become less aggressive reporting lower total values.

By contrast, higher volatility of intangibles increases the maximum prices in and out of the certified market. A greater volatility of intangibles is equivalent to a greater degree of information asymmetry. Since both larger and lower values of intangible assets become more likely ex-ante, then untruthful managers are able to claim larger values in a more credible way. Since the maximum prices in and out of the certified market are symmetrically affected by the increase in the volatility of intangible assets, certification decisions are (almost) unaffected by this change.

We then consider two extensions of the model. In the first extension, certification costs are modeled as the fee announced by a monopolistic certifier prior to the firm's certification decision- as in Lizzeri (1999). In that context we show that a higher volatility of tangible assets would not result in more certification (i.e., disclosure). The reason is that the certifier would accommodate any additional volatility by raising the fee exactly in the amount required to keep the probability of certification constant. A greater volatility is equivalent to a greater information asymmetry between the manager and the market. Disclosure/trading models (see Verrecchia, 1990 and Levin, 2001) have studied the impact of information asymmetry on the probabilitly of disclosure and trading: more information asymmetry has been often associated with more disclosure and with less trading. Our results show that this is not the case when certification fees (transaction costs) absorb the changes in information asymmetry that affect the market.

In the second extension we consider endogenous investment decisions. We assume that prior to selling the firm, the manager must incur an investment in order to generate the firm's total value –i.e., the sum of the firm's tangible and intangible assets. In this context, we study the impact of both the cost of certification and the volatility of tangible assets on the efficiency of the firm's investment decisions. First, consider the impact of certification costs. In the model, certification could improve the efficiency of investment by allowing a manager with low credibility to raise capital whenever good prospects arise. However, when the manager's credibility is sufficiently high, certification becomes a deadweight loss whose only role is to allow the manager to retain a greater portion of the trading surplus if tangible assets have high values. We show that expected certification expenses are single-peaked in the cost of certification. So after a certain point (precisely the one that maximizes the profits of the certifier) further increases in the cost of certification would actually benefit the manager from an ex-ante viewpoint: by strongly discouraging her tendency to certify ex-post, higher certification costs would lower expected certification expenses thereby raising the firm's ex ante value. Second, consider the effect of a higher volatility of tangible assets. Absent credibility problems, a greater volatility increases the firm's option value being thus beneficial for the firm's ex-ante value. However, when there are credibility problems, a higher volatility also boosts the demand for certification thus increasing certification expenses. For low levels of volatility and low levels of credibility the certification effect would dominate the option value effect: in that context, more volatility would be detrimental for the firm. Further, when certification costs are endogenous, more volatility would always be detrimental for the firm: that is, the option value benefits arising from higher volatility would always be fully offset by greater certification expenses, because more volatility would translate into a higher demand for certification as well as higher certification fees.

1.3 Literature Review

Perhaps our main contribution is to provide a general equilibrium-like theory of strategic communication where not only the message but also the *stage* of communication is at the sender's discretion. To understand this, a contrast with the disclosure literature might help. The disclosure literature studies the circumstances under which the sender unveils non-manipulable information to the public and how the public would interpret the sender's failure to disclose this information. Although useful in many real life context, this is a partial equilibrium analysis. In practice, failing to disclose does not imply that the sender would remain silent about his private information yet this failure may affect the way in which the sender manipulates information. Conversely, unveiling hard information does not mean the sender would mute himself on soft pieces of information that supplement his disclosure, yet it may alter how aggressively he manipulates this type of information.

In the standard model of disclosure (e.g., Milgrom 1981, Grossman 1981, Jovanovic 1982, Verrecchia 1983, Dye 1985, Shin 2003), the disclosure is always (i) fully credible and (ii) about the entire cash flow of the firm. Therefore, after a disclosure has been made, the firm is always perfectly priced by the buyers. But conditions (i)-(ii) exclude a number of situations of obvious interest for a practical implementation of these models. One, fully credible disclosures ("hard") are only a small part of all forward-looking information and, thus, these theories do not speak about non-certifiable components of information frequently disclosed by firms. Two, by assumption, the models assume that *all* managers are self-interested and would misreport any information that is not certified; without any initial cross-sectional differences in propensity to report high quality information, they do not provide testable predictions about inherent reporting quality, or links between reporting quality and the format or content of actual reports.

Our working model for information that is "soft," i.e. relies on the manager's propensity to disclose truthfully and is part of the cheap talk literature (Crawford and Sobel 1982, Gigler, 1994). This literature assumes that the sender's objectives are partly, but not completely, misaligned with those of the receiver. Like Sobel (1985), Benabou and Laroque (1992) and Morris (1992), we assume that the sender is privately informed about the extent of this misalignment. In particular, we assume that under certain circumstances (that only the sender observes) the sender is bound to tell the truth. The uncertainty of the receiver about the credibility of the sender together with the possibility to certify part of the information, enables the sender to partially overcome the receiver's skepticism, enabling informative communication even when incentives are likely to be extremely misaligned. By considering an alternative process to make the information hard and verifiable, the model allows us to measure the costs of lack of credibility and the alternative disclosure means that firms use to overcome these costs.

The unraveling principle was taken to a paradoxical extreme by the certification literature. Lizzeri (1999) for example showed that unraveling may occur even in the presence of extremely expensive certification fees and despite the certification technology being fully uninformative. When managers have no credibility, they are unable to communicate without the intermediation of an independent third party (i.e., a certifier.) In that context, the mere action to certify one's assets become a powerful signal, more relevant than the certification content itself (e.g., the rating). This endows a monopolistic certifier with great pricing power; actually more so the less accurate his rating technology become. We examine how a monopolistic certifier's rent extraction capacity is modified in the presence of some managerial credibility when the firm's value relevant information is only partially certifiable.

2 Model

This is a model of strategic communication between the seller of a good or service and its prospective buyers. For the purpose of this study, the good may be labor services, a product, or securities sold in a financial market but, in order to facilitate the exposition, we use the interpretation of a firm whose stock is traded in a competitive financial market. We refer to the seller as the manager, to the buyers as the market and to the item as the firm. The firm is sold in a competitive market and the manager maximizes the resale price of the firm.

Let π be the value of the firm, and assume that this value may depend on two pieces of information that are privately observed by the seller, a "hard" piece h that the manager can certify and a "soft" piece s that the manager can only communicate to the market informally.

A.1. The value of the firm is additively-separable in the hard and soft information, i.e. $\pi = h + s$.

The purpose of the additive structure is to consider a research design in which soft information does not increase or decrease the value of assets that could be certified. The assumption guarantees that linkages between certification and soft signals are entirely driven by informational asymmetries.

A.2. The cost of certifying h is $c > 0$, in which case h is always truthfully disclosed by the manager.

One can think of certification as any process through which an independent party attests about the value of h, such as hiring an auditor to verify financial statements or using the services of a credit rating agency.[1] The cost may also represent competitive cost involved in disclosing proprietary information about those assets to make the disclosure verifiable to outside observers. The manager always has the option not to certify, in which case both h and s, and thus π, are disclosed as *soft* information, i.e., information that can be potentially misreported by the manager. What we call a non-certification could be thought of as the manager choosing to disclose in a press release, or providing an aggregate number with little supporting (proprietary) evidence that the disclosure is appropriate.

A.3. h and s are independently distributed and satisfy: (i) $E(h) = E(s) = 0$, (ii) $h = \sigma\omega$ where $\sigma > 0$ and ω has a log-concave distribution $F(\cdot)$ with positive density $f(\cdot)$ over $[-1, 1]$, (iii) s has a binary distribution with support over $\{-q, q\}$.

The manager's truthfulness is represented by a binary random variable $\tau \in \{0, 1\}$, where $\tau = 1$ indicates that the manager must report truthfully any private information he is aware of and $\tau = 0$ indicates that the manager has reporting discretion over any information that is not certified. In other words, when $\tau = 0$, any uncertified message is pure cheap talk.

A.4. The manager's truthfulness $\tau \in \{0, 1\}$ is independent from h and s and is such

[1] As Lang and Lundholm (1993) point out, the notion that preparation costs have a fixed component underlies much of FASB's and SEC's consideration of firm size in disclosure requirements. The authors mention that the SEC has separate 10K and 10Q filing requirements for small firms, labeled 10KSB and 1OQSB, to lighten the burden of accessing the equity markets.

that $\Pr(\tau = 1) = \gamma$. The realization of τ is known to the manager but not to the buyers.

We refer to $\theta = \frac{\gamma}{1-\gamma}$ as the firm's credibility, i.e., the likelihood that the manager is forced to tell the truth. θ could capture things such as the manager's honesty or ethical standards, the quality of the firm's control system, the effectiveness of market institutions, the existence of incentives to maximize interim stock prices, etc. The market is competitive, and the firm is priced at its expected value conditional on all publicly available information.

A.5. The uncertainty about soft information is sufficiently large, i.e. $q \geq \underline{q}$ (where $\overline{q}$ is defined in Appendix B).

We focus here on the case in which soft information plays an important role relative to hard information. This assumption provides more tractability to the model and seems reasonable for our main application given that, in the vast majority of cases, the value of a firm depends more highly on forward-looking intangibles (e.g., brand images, research projects, etc.). From a conceptual standpoint, the assumption is useful to remove less interesting cases in which soft information is a second-order effect and thus where the analysis would be very similar to a single-dimensional costly certification.

The time-line of the model contains the following events. First, the manager privately observes reporting discretion τ, and the realization of hard and soft information h and s. Second, the manager decides whether to (i) certify h for a cost c and make an informal report about s, or (ii) not certify any information and report the entire value π informally. Third, upon observing the manager's report and certification choice, the buyers compete to buy the firm and the price of the firm's assets is set equal to the expected value conditional on all public information.

2.1 Beliefs, Strategies and Equilibrium Definitions

A Perfect Bayesian Equilibrium (PBE) consists of a reporting strategy, a price system after any possible report and a certification strategy. These objects are formally defined below.

Reporting Strategy In general, one can think of the manager's message as a two-dimensional report about (h, s) –plus a binary certification decision $d : \{0, 1\} \times \mathbb{R}^2 \rightarrow \{0, 1\}$, where $d = 1$ means that h is certified– satisfying that

(a) conditional on $\tau = 1$, the report must be truthful, and

(b) conditional on $\tau = 0$:

i. if the manager chooses to not certify h, then she can report anything $-$her report must only belong to the support of (h, s);

ii. if the manager chooses to certify h then h becomes publicly observable, but she can still lie about the soft component.

It is convenient (and without loss of generality) to simplify the description of the report by assuming that when the manager does not certify h she reports the total value π and when she does certify h she only reports the value of s (as the value of h becomes public under certification). For future use, we define $f_{\pi}(.)$ as the density of $h + s$.

When $\tau = 0$, r_0 denotes the manager's report under no certification and r_1 denotes the manager's report under certification. The manager may choose to randomize her reports. We represent this randomization by two functions $\varphi_0(\cdot)$ and $\varphi_1(\cdot)$. The former is the p.d.f. of r_0 and the latter is the probability mass function of r_1. In the following, we refer to φ_0 and φ_1 as the manager's reporting strategies - recall that under no discretion the seller has no choice but truth-telling.

Price System The investors' information set I may be either $I = \{d = 0, x_0\}$ or $I = \{d = 1, h, x_1\}$, where $x_0 = r_0$ if $\tau = 0$ and $x_0 = \pi$ if $\tau = 1$. Similarly, $x_1 = r_1$ if $\tau = 0$ and $x_1 = s$ if $\tau = 1$.

The price $P(I)$ is then defined by the following conditional expectation

$$P(I) = E(\pi|I) - cd$$

Lastly, since the untruthful manager chooses the report to maximize $P(I)$, the relevant price for that manager is the maximal price that can be achieved conditional on the chosen certification. Accordingly, we define $p_0 = \sup_{x_0} P(\{d = 0, x_0\})$ and $p(h) = \sup_{x_1} P(\{d = 1, h, x_1\})$.

Equilibrium Definition We use the equilibrium concept of Perfect Bayesian ("PBE"), as defined below. The manager's private information is denoted $\omega = \{\tau, h, s\} \in \Omega$.

Definition 1 *A PBE consists of certification strategies, $d(\cdot) : \Omega \to \{0, 1\}$; reporting strategies for the untruthful manager, $\varphi_d(\cdot) \in \Delta$; and a pricing function, $P(\cdot)$, such that:*

(a) Given $\tau = 0$; (h, s) and $P(\cdot)$, the manager's reporting and certification choices, $\{d, \varphi_d\}$, maximize $P(I)$.

(b) Given $\tau = 1$; (h, s) and $P(\cdot)$, d maximizes $P(I)$ subject to $x_d + dh = \pi$.

(c) On the equilibrium path the pricing function is computed according to Bayes' rule as

$$P(I) = E(\pi|I) - cd.$$

Most of the elements of the equilibrium are standard. Condition (a) states that the untruthful manager certifies and reports optimally. Condition (b) states that the truthful manager certifies optimally and reports truthfully. Condition (c) states that prices are computed according to Bayes' rule conditional on conjectured certification and reporting strategies.

As in any signalling game, there may be more than one equilibrium. Indeed, the equilibrium definition is silent about the way prices are formed off the equilibrium path. In Section 4 we will show that a simple refinement guarantees the existence of a unique equilibrium with positive probability of certification.

3 Equilibrium without certification

We develop a benchmark type of equilibrium in which no firm certifies regardless of what information is observed. This will have two main purposes: first, to illustrate how soft information may alter prior findings in the disclosure literature and, second, to lay out a simplified outline of a more general argument that will be used in later sections.

The first step is to derive characteristics of the pricing function when information is not certified, based on beliefs that no firm certifies (which we shall confirm later). Let us observe that the untruthful managers will always make reports r_0 that maximize the market price, and thus we can denote as $\hat{p}$ the market price that is attained in these cases. Any report x_0 strictly below $\hat{p}$ would not have been made by the untruthful manager and thus should be viewed as entirely credible, i.e. $P = x_0$. We thus make the following observation:

$$P(\{d = 0, x_0\}) = \min(x_0, \hat{p}) \tag{1}$$

The price $\hat{p}$ must be consistent with Bayesian updating. Specifically, any report $x_0 \geq \hat{p}$ may have been issued by a truthful manager with $\pi \geq \hat{p}$, or an untruthful manager who always reports $x_0 \geq \hat{p}$ regardless of the information received; in the latter case, the untruthful manager will generate (in expectation) zero value. These observations lead to the following condition.

$$\hat{p} = \theta E\left(\pi - \hat{p} | \pi > \hat{p}\right) \Pr\left(\pi > \hat{p}\right) \tag{2}$$

Lemma 2 *Equation* (2) *has a unique solution* $\hat{p} \in (0, q + \sigma)$.

Lemma 2 indicates that, in an equilibrium with no certification, the soft disclosure is not viewed as entirely credible if it is a sufficiently large, above-average report r_0. Note that all disclosures of negative outcomes are always viewed as credible and, for such events, certification would serve no purpose.

To verify that no certification is indeed an equilibrium, we need to establish that no firm would be willing to certify. The firm which would have been the most willing to certify is one with $h = \sigma$ (the most favorable information) and, if it did certify and report $s = q$, the off-equilibrium market price may be $p' \in [\sigma - q - c, \sigma + q - c]$, depending on how the market perceives the value the soft information of a certifying firm. It follows that no certification is an equilibrium if and only such p' can be found less than $\hat{p}$, i.e. such that $\hat{p} \geq \sigma - q - c$, as stated next.[2]

Proposition 3 *An equilibrium with no certification always exists. The reporting strategy is given by:*

$$\varphi_0\left(r_0\right) = \theta \frac{r_0 - \hat{p}}{\hat{p}} f_\pi\left(r_0\right) \text{ for } r_0 \in [\hat{p}, \sigma + q] \tag{3}$$

The firm's maximum price $\hat{p}$ *increases in* θ, q *and* σ.

The reporting strategy (of the untruthful manager) in an equilibrium with no certification has an intuitive interpretation. For any $r_0 \geq \hat{p}$, the true reporting density $f_\pi(\cdot)$ is altered by two terms. The first term θ represents the ex-ante credibility of managers and means that, as managers are perceived as more truthful, the untruthful managers report more aggressively all values $r_0 \geq \hat{p}$. The second term $(r_0 - \hat{p})/\hat{p}$ represents an additional distortion for high reports. That is, *even though these reports yield the same price in equilibrium,* the untruthful manager reports relatively more high reports r_0 than the truthful manager. In particular, the odds of an untruthful manager increase when observing a higher report r_0.

The no-certification equilibrium exists even if the certification is entirely costless or $c = 0$, in contrast to the unravelling theorem (see, e.g., Grossman, 1981, and Milgrom,

[2] The interpretation of a price $p' = h - q - c$ should be that, conditional on certifying h and reporting s, the market believes that this off-equilibrium message has been issued by an untruthful firm with $(h, s) = (h, -q)$.

1981). The reason for this is that certification alone does not confirm the entire value of the firm but may imply adverse beliefs about the remaining soft assets of the firm. As ex-ante credibility θ increases, the maximum uncertified price $\hat{p}$ and, therefore, the no-certification equilibrium will exist for a wider range of beliefs.

4 Equilibrium with certification

4.1 Refinement

The equilibrium without certification is predicated on investors believing that untruthful managers are more likely to send an off-equilibrium message.[3] Yet, these beliefs are partly arbitrary and, in a similar manner, could lead to many other possible equilibria. We introduce the following equilibrium refinement based on the possibility of trembles by the manager.

Definition 4 *A perturbed game is a game in which the manager makes a certification error with probability* $\varepsilon > 0$.

For example, in a perturbed game the manager may end up certifying h when certification was not optimal. Note that in a perturbed game any combination of report and certification choice has positive probability, hence investors' beliefs are always determined by Bayes' rule in the perturbed game.

Refinement (R) *An equilibrium in the original game is robust if it satisfies: (a)* φ_0 *is independent of* π *and* φ_1 *is independent of* s*; (b) the equilibrium can be obtained as the limit, when* ε *tends to zero, of a perturbed equilibrium.*

R (a) simply states that the untruthful manager should not condition her reports on information that markets will never be able to verify. Put differently, the manager's reports should not depend on observations that are payoff irrelevant –as in sun-spot equilibria. This condition is sufficient to rule out the possibility of equilibria in which the behavior of untruthful managers induce discontinuities in the pricing function.

R (b) is similar in spirit to the notion of trembling hand perfection (Selten, 1975) or sequential equilibrium (Kreps and Wilson, 1982). Observe that this perturbation is the simplest way to ensure that all possible combinations of report/certification lie on the

[3] One can show that these beliefs are robust to the intuitive criterion and the Bank and Sobel's D1 criterion.

equilibrium path. In the sequel we study PBE that satisfy R, which we refer to as robust equilibria.

4.2 Analysis of the Equilibrium

We develop here some preliminaries required for a complete analysis of certification decisions. Many of these preliminary results also provide key conceptual results for the intuitions of the model and, therefore, we shall spend some time here explaining their rationale. We adopt the convention that the manager certifies when indifferent between certification and no certification.

Proposition 5 *The pricing function is given by:*

$$P(\{d=0,x_0\}) = \min(p_0,x_0) \tag{4}$$
$$P(\{d=1,h,x_1\}) = h-c+\min(x_1,z) \tag{5}$$

where $p_0 \geq 0$ *and* $z = \frac{\theta}{1+\theta}q$.

Proposition 5 is very intuitive for reports that are made on the equilibrium path. When buyers observe a report that would not lead to the maximal price, i.e. less than p_0 in the uncertified market or $h-c+z$ in the certified market, they infer that such a report must have been made by a truthful manager, and thus price the firm according to the report, i.e. $P(\{d=0,x_0\}) = x_0$ and $P(\{d=1,h,-q\}) = h-c-q$. In other words, a sufficiently low soft report is always fully credible. When a report consistent with the maximal price is issued, buyers do not fully trust the report and a lower price than that which would have prevailed under truthful reporting is offered. In the special case of a certification, the maximal soft report $r_1 = q$ could have been issued by either a truthful manager or an untruthful manager: this leads to a price for the soft signal that is between zero (entirely untruthful) and q (entirely truthful).

We turn next toward the optimal certification strategy. Since certification is costly, it would only be chosen if (1) the report is not entirely credible in the uncertified market, i.e. it would imply a price p_0, (2) the price with certification is greater than p_0. For the truthful manager with $s=-q$ (for which certification always imply perfect credibility), these conditions can be written as: $h-c-q \geq p_0$. For the truthful manager with $s=q$ (for which certification never implies credibility), these conditions can be written as: $h-c+z \geq p_0$. Lastly, note that the untruthful manager would always report in the same manner as the truthful manager conditional on certifying ($r_1 = q$) and would always attain p_0 conditional on not certifying. Thus, the condition for certification is the same as for the truthful manager with $s = q$.

Proposition 6 *There exists $\mathbb{H} = (H_q, H_{-q})$, where $H_q \leq H_{-q}$, such that the truthful manager certifies if and only if $h \geq H_s$ and the untruthful manager certifies if and only if $h \geq H_q$.*

This pricing function has a few other implications regarding the reporting strategy adopted by the untruthful manager. First, because $h - c + z > h - c$, the untruthful manager should report high soft information when certifying, i.e. $r_1 = q$, in order to avoid being perceived as a manager with bad soft information. Second, when not certifying, the untruthful manager should report r_0 in a manner consistent with the prices, i.e. $r_0 \in [p_0, H_q + q)$. This implies in particular that, in an equilibrium where certification may occur, the highest possible report $\pi = \sigma + q$ is *never* issued without a certification.

Another important implication of the pricing function is that the uncertified price must be constant on $[p_0, H_q + q)$. Given that a higher value indicates higher price if the manager is truthful, such a constant price may only occur if buyers view a higher report as more likely to have been issued by an untruthful manager. That is, (perceived) credibility decreases as the manager reports higher total value. The next Proposition indicates the equilibrium reporting strategy on this interval.

Proposition 7 *In an equilibrium such that a manager with $s = q$ may certify,*

$$\varphi_0(r) = \theta \frac{r - p_0}{p_0 - E(\pi | h < H_q)} \frac{f_h(r - q)}{2F_h(H_q)} \text{ for } r \in [p_0, H_q + q] \tag{6}$$

All other things equal, more anticipated certification leads to a more concentrated set of reports. Furhter, since more anticipated certification results in the market perceiving uncertified firms in a more negative way, the untruthful manager's reporting behavior becomes less aggressive: on average she reports lower values (i.e., $E(r_0)$ goes down.)

4.3 Equilibrium

Having written all variables in terms of p_0, we are now left to examine which values of p_0 could be consistent with Bayes' rule.

Proposition 8 *There is a unique robust equilibrium characterized as follows. The truthful manager with $s = -q$ does not certify (i.e., $H_{-q} \geq \sigma$). The truthful manager with $s = q$ and the untruthful manager certify if and only if*

$$h \geq H_q = \min(k, \sigma).$$

The untruthful manager reports $r_1 = q$ and r_0 as given by Equation (6)*. The maximal non-certified price is given by:*

$$p_0 = \min(k, \sigma) - c + z \tag{7}$$

where k is the unique solution to

$$k - c = E(h|h \leq k). \tag{8}$$

The limit cases provide a useful overview of the equilibrium's basic properties. Consider first the effect of θ. When $\theta \to 0$, then $z = E[s]$ and $p_0 = \min(k, \sigma) - c$. In that case, the market expectations would only be affected by whether or not the firm certifies h but they would not be affected by the manager's report. Put differently, when there is no credibility, only actions should affect expectations. The other extreme, when $\theta \to \infty$, requires that $q \to \infty$ to satisfy A.5. However, it is easy to see that, the effect of $\theta \to \infty$ is to remove the upper bound on the price of uncertified firms so that $p_0 \to q + \sigma$. Of course, in such equilibrium, certification is unnecessary because the credibility problem that stimulates the need for certification vanishes. Consider now the limit cases in terms of c. When $c = 0$, the price, given no certification, becomes $p_0 = z - \sigma$. Thus, the untruthful manager always certifies h, because if he failed to certify h the market would assume the worst possible scenario about h. Interestingly, for $\theta > 0$ there is no unraveling: the truthful manager does not certify when $s = -q$.

When $c \to \bar{c}$ certification shuts down ($k \to \sigma$.) When this happens, the maximum price of an uncertified firm would naturally converge to $\hat{p}$. The following corollary studies the comparative statics for p_0, z and H_q.

Corollary 1 *(i) The maximum price of uncertified firms, p_0, increases in θ, c but decreases in σ.*

(ii) The maximum price of certified firms, $p(h)$, increases in h, q and θ.

(iii) The certification threshold, H_q, increases in c, but decreases in σ.

All these comparative statics are intuitive. The integrity of the firm, θ, has a positive effect on the firm's maximal prices p_0, z. The effect of θ on z is almost mechanical.

Holding constant both certification and reporting strategies, an increase in θ raises the probability of truth-telling conditional on the manager reporting good news. We refer to this as the credibility effect. This effect allows z to go up because z is precisely the price of s arising when the manager reports good news about s.

A similar effect is behind the positive association between p_0 and θ. A higher θ, raises the credibility of reports over the right tail of the distribution of x_0, that is it raises the credibility of good news. However, when A.5 is violated two additional effects may be present: a *certification* and a *fraud* selection effect. First, consider the certification effect. Increasing θ induces a lower propensity to certify information for both $\tau = 0, 1$. As some marginal firms opt-out from the certified market, the average value of uncertified firms surges because marginally certified firms are on average more valuable than uncertified firms. Thus the certification effect reinforces the credibility effect towards increasing p_0. Second, consider the fraud effect. The fraud effect, refers to how θ, indirectly, affects the chances of misreporting in the uncertified market by altering the relative propensity of certification of the untruthful manager. An increase in θ, could in principle lead to a stronger concentration of misreporting in the uncertified market undermining the credibility of uncertified reports. If that happened, the fraud effect would go against the credibility effect because misreporting firms have lower values (relative to the price they claim.) However, under A.5, the fraud effect is not present because the relative likelihood of misreporting ($\frac{\Pr(d=0|\tau=0)}{\Pr(d=0|\tau=1)}$) in the uncertified market is independent of θ.

In fact, under A.5, the integrity of the firm θ does not alter the certification threshold, H_q. The reason is that the maximum prices with and without certification (i.e., $p(h)$ and p_0) are affected in the same manner by θ, thus the threshold, $H_q = p_0 + c - z$, capturing the manager's propensity to certify h does not vary in θ. However, when A.5 does not hold, p_0 tends to be more sensitive to θ than $p(h)$, so that H_q increases in θ. This reinforces the idea that the probability of certification goes down as θ increases.

Now consider the effect of c. The cost of certification, c, raises the maximum price of uncertified firms p_0 because of certification selection: the increase in c makes certification unaffordable for some marginal firms (to see this, note that certification threshold, H_q, goes up as c increases). As some firms switch to the uncertified market, the average value of uncertified firms increases. This effect is only partially offset by the fraud selection effect: the frequency of frauds in the uncertified market goes up in c (as we discuss below).

Observe that the value of z is independent from both c and h. This is perhaps surprising once we recognize that the credibility of a certified report, $\Pr(\tau = 1|h, d = 1)$, depend on the value of c and h. In fact both c and h affect the manager's certification choice differently depending on τ, potentially affecting the way the market perceives the credibility of a certified firm. For example, the market could perceive as more likely that a certified report entailed discretion when the cost of certification is too high–as opposed to when the cost of certification is too low. Similarly, the market could perceive as relatively more likely that a certified report entailed discretion when the value of h is relatively low. This might be true but does not affect the determination of z: even though the probability of discretion conditional on certification, $\Pr(\tau = 1|d = 1, h)$, does depend on both c and h, the probability of discretion conditional on certification and the manager's report $\Pr(\tau = 1|d = 1, h, x_1)$ does not. The latter probability only depends on the likelihood that the manager claims x_0 with discretion relative to the likelihood that he claims x_0 without discretion.[4]

The effects of σ and q are also intuitive. Recall that these parameters represent the uncertainty about hard and soft information. Increasing the uncertainty of soft information, q, raises both p_0 and z. Consider the effect on z: increasing q raises the expected value of the firm conditional on good news about soft information (other things equal). Since z is the price of s that follows when the manager reports good news about s then z must increase in θ. Indirectly, the same effect explains the positive association between p_0 and q. Whenever the manager induces a price p_0 in the uncertified market, he (implicitly) is reporting good news about s, because, under $\tau = 1$, the manager is more likely to issue a report above p_0 in the uncertified market when $s = q$.

Consider the effect of σ. Clearly, σ plays no role in the certified market, where the uncertainty about h is perfectly resolved via certification. But it does play a role in the uncertified market. Note that a higher σ diminishes the value of the firm conditional on bad news about h. Since, no certification is interpreted as an indication of bad news about h, then the value of an uncertified firm should decrease in σ. This is what drives p_0 down as σ increases: a greater σ implies a lower expected value conditional on bad

[4] To see this note that in equilibrium

$$\begin{aligned}
\Pr(\tau = 1|d = 1, h, x_1) &= \frac{\Pr(d = 1, h, x_1|\tau = 1)\,\gamma}{\Pr(d = 1, h, x_1|\tau = 1)\,\gamma + (1-\gamma)\Pr(d = 1, h, x_1|\tau = 0)} \\
&= \frac{f_h(h)\, f_s(x_1)\,\gamma}{f_h(h)\, f_s(x_1)\,\gamma + (1-\gamma)\Pr(d = 1, h, x_1|\tau = 0)} \\
&= \frac{f_h(h)\, f_s(x_1)\,\gamma}{f_h(h)\, f_s(x_1)\,\gamma + (1-\gamma)\, f_h(h)\,\varphi_1(x_1)} \\
&= \frac{f_s(x_1)\,\gamma}{f_s(x_1)\,\gamma + (1-\gamma)\,\varphi_1(x_1)}
\end{aligned}$$

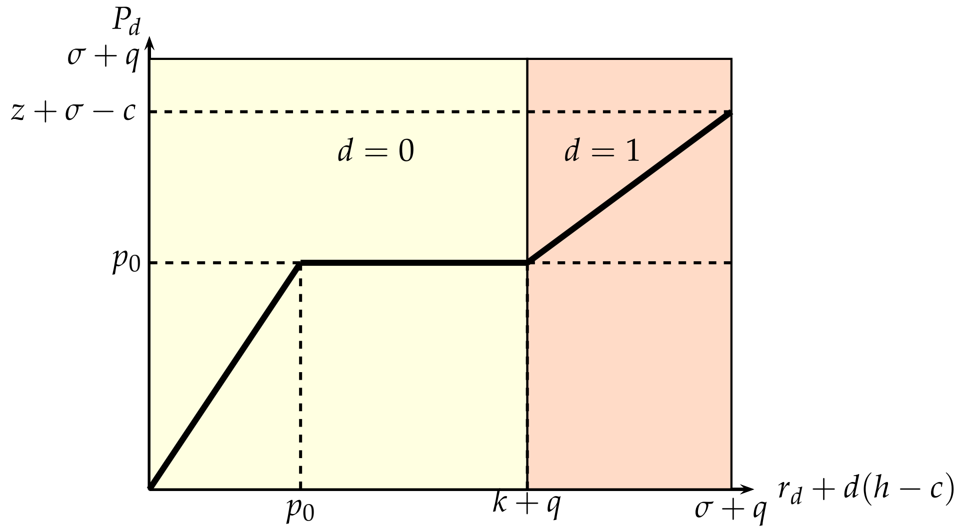

Figure 1: Prices

news, thus a lower expected value conditional on no certification.

In the following we study in greater depth three properties of the equilibrium: the likelihood of certification, the likelihood of misreporting, and the magnitude of misreporting.

4.4 The likelihood of certification

As mentioned previously, the propensity to certify information is especially strong when the manager is untruthful because untruthful managers experience stronger credibility problems and certification is their only tool to overcome market's disbelief. The extent to which the manager uses this tool depends on a trade-off between the credibility benefits of certified reports and certification costs. This trade-off is captured in the next corollary.

Corollary 2 *The probability of certification decreases in credibility, θ, decreases in the cost of certification, c, and increases in the uncertainty of hard information, σ.*

An increase in θ reduces the need to certify information by making the firm more credible ex-ante, both with and without discretion. This is natural in our model but goes against the way certification is often viewed, that is as a sign of transparency (see e.g. Botosan, 1997). In our model, a high propensity to certify information reveals a credibility problem (a low θ) rather than a high level of transparency. The reason that we observe certified disclosures is because there is a credibility problem in the first place.

An increase in the uncertainty of hard information, σ, results in a higher probability of certification because it lowers the price of uncertified firms, p_0, thus raising the penalty from failing to certify h. The effect of c is even more direct. A higher c discourages certification by uniformly reducing the surplus the manager can obtain via certification. Note that despite its intuitive appeal, this effect does not hold in the corner equilibrium where the direct effect of an increase in c is offset by the fact the fact that the lack of certification is more strongly penalized by markets when c increases.

4.5 The likelihood of misreporting

A related question is whether misreporting is more likely in the certified or in the uncertified market. To answer this question it suffices to note that the manager is more likely to certify information when discretion is available. Hence, misreporting must be more likely in the certified market. The difference though is that in the certified market, misreporting affects only soft information whereas in the uncertified market misreporting affects both soft and hard information. Also,

Corollary 3 *(i) The probability of misreporting in the certified market decreases in θ.*

(ii) The probability of misreporting in the uncertified market decreases in θ and σ but increases in c.

The integrity of the firm affects the probability of misreporting in both markets in the obvious way (reducing it) so we will omit discussing this. In the uncertified market, the reason that the probability of misreporting increases in c lies in the fact that the probability of no certification is relatively more sensitive to c when $\tau = 0$ as opposed to when $\tau = 1$. Recall that in the absence of discretion, the manager never certifies information when $s = -q$, regardless of the value of h. Conditional on a low s, his decision is thus not affected by c. By contrast, under discretion the manager always reports good news about s.Therefore his decision to not certify information is always affected by c, even when soft information is bad. As a result, an increase in c lowers the probability of no certification under $\tau = 0$ in a stronger fashion than it lowers the probability of no certification under $\tau = 1$. This is why the chances of misreporting increase in c in the uncertified market.

The same reason explains why the probability of misreporting in the uncertified market decreases in σ. A greater σ increases the overall probability of certification, but it does so more strongly when the manager has discretion. By reducing the probability of discretion in the uncertified market, a greater volatility of hard information results in a lower probability of misreporting in the uncertified market.

4.6 The magnitude of misreporting

Perhaps as important as the frequency of misreporting, is the magnitude of misreporting. After all, the harshest financial regulations seem to be driven by the discovery of large frauds (large overstatements). We define the expected magnitude of frauds in market d as follows

$$F_d = E\left[r_d\right] - E\left[\pi | d, \tau = 0\right]. \tag{9}$$

Hence, F_d measures the average overstatement incurred by the manager under discretion in market $d \in \{0, 1\}$.

Corollary 4 *(i) The average magnitude of overstatements in the uncertified market,*

$$F_0 = q + \frac{\theta + 2}{2q} Var\left(h | h < k\right),$$

increases in θ, c and q.
(ii) The average magnitude of overstatements in the certified market,

$$F_1 = q,$$

increases in q.

We can draw three main lessons from this corollary. First, the magnitude of frauds is particularly large when integrity, θ, is high. Apparently counterintuitive, this result is natural. What stimulates large overstatements is precisely the trust that markets assign to financial reports. But a high level of trust is possible only if the firm's propensity to misreport information is low, i.e., if the firm has a high level of integrity.

Second, the cost of certification c increases the magnitude of reports in the uncertified market without affecting the magnitude of reports in the certified market. Higher certification costs induce a first order stochastic increase in the distribution of r_0. This is due to the certification selection effect. As certification becomes more expensive, some marginal firms move towards the uncertified market, leading to an increase in the true value of the uncertified firms that claim a value above p_0. This allows the manager to report higher values when discretion is available, when $\tau = 0$. This does not by itself imply that F_0 increases in c, as there is another effect going in the opposite direction. The true value of the firms misreporting information in the uncertified market ($E[h|h<k]$) increases in c (because k increases in c.) However, this is only a second order effect as compared with the effect of c on $E[r_0]$.

Third, in the certified market, the volatility of information tends to increase the magnitude of overstatements. In particular, a higher volatile of soft information, q, almost mechanically increases F_1. More generally, when s has a continuous distribution, any increase in the risk of s leads to larger overstatements in the certified market. More risk simply increases the likelihood of the tails of the distribution of s ex-ante, thus increasing the credibility of s falling in the right tail too. The effect of σ on the magnitude of frauds is less clear. For a fixed k, a higher σ would increase $Var(h|h<k)$ thus leading to a larger F_0. However, a larger σ would also lead to more certification and thus to a lower value of k, which decreases F_0. Another way to see this is to consider that increasing σ reduces the price of uncertified firms, thus reducing the average magnitude reported by the manager in the uncertified market when $\tau = 0$, but it also decreases the true value of misreporting firms in the uncertified market.

5 Endogenous certification fee

Here we endogenize the value of c. In particular, we assume that prior to the release of the report, a monopolistic certifier publicly announces a non contingent fee c which the firm must pay for the certification of h. When choosing c, the certifier ignores the actual value of $\{h, s, \tau\}$ but is aware of its distribution. If hired, the certifier is able to learn the exact value of h at not cost. The certifier's independence is out of question so, if hired, the certifier truthfully reveals the value of h to the market (unlike in Lizzeri (1999), here the certifier is not allowed to choose a noisy certification technology.)

The certifier maximizes expected profits, Π, so that c^* is defined as

$$c^* = \arg\max_{\hat{c}} \Pi \equiv \Pr(d=1|\hat{c}) \times \hat{c}.$$

so that the certifier's expected demand is given by the probability of certification.

Corollary 5 *(i) There exists a unique certification equilibrium in which*

$$c^* = \arg\max_{\hat{c}} \left[1 - F_h\left(k\left(\hat{c}, \sigma\right)\right)\right] \hat{c}.$$

(ii) The optimal fee c^* increases in σ but is independent of θ.

(iii) The probability of certification $\Pr(d=1|c^*)$ *is independent of* σ.
(iv) The certifier's profits,

$$\Pi^* = \frac{\theta/2+1}{1+\theta}\left[1-F_h\left(k\left(c^*,\sigma\right)\right)\right]c^*,$$

increase in θ *and* σ.

The certifier's expected profits are negatively affected by the integrity of the manager but favorably affected by the volatility of hard information σ. This is because, a higher θ decreases the demand of the certifier whereas an increase in σ increases it. Note that given our distributional assumptions, the certifier's optimal fee c^* does not depend on the integrity of the firm θ, which only has the effect of scaling down the expected profits of the certifier. Also note that c^* is such that the probability of certification is independent of the uncertainty of hard information σ. The certifier exploits a greater uncertainty about h by charging higher fees, so that in equilibrium the actual probability of certification does not vary in σ. This results contradicts a basic prediction of the disclosure literature (see e.g., Verrecchia, 1990) asserting that greater information asymmetry between the manager and the market (represented by a larger σ) would induce more disclosure (i.e., a higher probability of certification.)

6 Investment efficiency

Suppose now that in order for the firm to realize the firm's terminal value π, the manager must incur an investment K, prior to selling the firm. To sell the firm, the manager must choose one of two options. He could either report π in the uncertified market, in which case the firm's price would depend on the credibility of her report. Alternatively, the manager could appeal to the certified market market, where the value of h would be certified at a cost c.

For simplicity, we assume that

$$K > \sigma - q,$$

so that investing is never (socially) optimal when soft information is unfavorable. To consider the most interesting case we also assume that

$$p_0 > K, \tag{10}$$

so that the uncertified market is feasible. That is, in principle (if the manager's report is sufficiently high) the firm can be sold in the uncertified market. Implicitly, this condition is setting a lower bound on θ.

We would like to understand how the integrity of the firm and the certification cost c affect the investment efficiency EF as represented by the expected trading surplus. We represent trade by a dummy $T \in \{0, 1\}$ where $T = 1$ means that the firm is sold –so that investment actually takes place. Thus, EF can be written as

$$EF = \Pr(T = 1) [E(\pi - K - cd | T = 1)] \tag{11}$$

Corollary 6 *There exists a unique investment equilibrium in which*

$$T = \begin{cases} 1 & \text{if } \{\{\tau = 0\} \cup \{\tau = 1, \pi \geq K\}\} \\ 0 & \text{otherwise} \end{cases} . \tag{12}$$

In such equilibrium, EF is given by

$$EF = -\frac{K + cF_h(-k)}{1 + \theta} + \frac{\theta}{1 + \theta} \frac{\int_{K-q}^{\sigma} (h + q - K)\, dF_h(h) - cF_h(-k)}{2} \tag{13}$$

$$= \frac{\theta}{1 + \theta} \frac{\int_{K-q}^{\sigma} (h + q - K)\, dF_h(h)}{2} - \frac{K}{1 + \theta} - \Pi. \tag{14}$$

We see that there are two sources of inefficiency. First, under discretion, the manager always invests and sells the firm even when the project has negative NPV, because he can always overstate its NPV in the uncertified market –by contrast, under $\tau = 1$, the manager only invests when the firm's NPV is positive. The second inefficiency is related to certification: the manager incurs in costly certification of h as a means of retaining a greater share of the surplus. This expense is of course socially inefficient.

Corollary 7 *(i) If $c \geq c^*$, efficiency increases in the cost of certification c, and vice versa.*
(ii) If $c = c^*$, then EF always decreases in σ.

Consider the effect of c. An increase in certification costs uniformly reduces the return from investing in the certified market (recall that in the certified market, carrying out the investment requires $K + c$ rather than only K.) If the probability of certification was fixed, a greater c would reduce the average return of investment, thus it would reduce EF. However, the increase in c generates also another effect going in the opposite direction that is particularly strong when $c \geq c^*$. Increasing c reduces the probability of certification thereby alleviating the inefficiency that arises from the manager's tendency to certify h. In fact, when $c \geq c^*$ increasing c may strongly reduce certification so that the expected certification expense, Π, is reduced after the increase in certification costs c. This result was implicit from Section (6). Note that the expected certification expense, corresponds to the certifier's profits Π in Section (6), which are maximized when $c = c^*$.

Consider the effect of σ. Again, there are two opposed effects. First, there is the option value effect: a greater σ increases the firm's option value ex-ante. Second, there is the certification effect: a higher σ increases the propensity of the manager to certify σ leading to higher certification expenses. This effect dominates the option value effect when σ is small enough or θ is low. Furthermore, when c is endogenous, an increase in σ is always detrimental to the manager, ex-ante.

We illustrate this effect in the following example.

Example 9 *Assume h is uniformly distributed over $[-\sigma/2, \sigma/2]$. Then, in equilibrium $k = 2c - \sigma/2$, and thus*

$$\begin{aligned} EF &= \frac{\theta}{1+\theta} \frac{\int_{K-q}^{\sigma/2} (h+q-K) \frac{1}{\sigma} dh}{2} - \frac{K}{1+\theta} - \left(\frac{1}{1+\theta} + \frac{\theta/2}{1+\theta} \right) \left(1 - \frac{2c}{\sigma} \right) c \\ &= \theta \frac{(\sigma/2 + q - K)^2}{(1+\theta) 4\sigma} - \frac{K}{1+\theta} - \underbrace{\frac{1}{2} \frac{2+\theta}{1+\theta} \frac{\sigma - 2c}{\sigma} c}_{=\Pi} \end{aligned}$$

Differentiating with respect to σ yields

$$\frac{\partial EF}{\partial \sigma} = \frac{1}{16} \frac{\theta\sigma^2 - 4\theta K^2 + 8\theta K q - 4\theta q^2 - 32c^2 - 16\theta c^2}{(1+\theta)\sigma^2} \tag{15}$$

which is negative if and only if

$$\sigma \leq 2\sqrt{(K-q)^2 + 4c^2 \frac{2+\theta}{\theta}}$$

Remarkably, when c is endogenous $\frac{\partial EF}{\partial \sigma}$ is always negative, so the certification effect always dominates the option value effect. In fact,

$$\Pi \equiv \frac{1}{2} \frac{2+\theta}{1+\theta} \left(1 - \frac{2c}{\sigma} \right) c$$

which yields

$$\Pi^* = \frac{1}{8}\sigma,$$

at

$$c^* = \frac{1}{4}\sigma.$$

Substituting this into (15) *yields*

$$EF = \theta\frac{(\sigma/2 + q - K)^2}{(1+\theta)\,4\sigma} - \frac{K}{1+\theta} - \frac{1}{2}\frac{2+\theta}{1+\theta}\frac{1}{8}\sigma,$$

Thus

$$\frac{\partial EF}{\partial \sigma} = -\frac{1}{8}\frac{2\theta\,(K-q)^2 + \sigma^2}{(1+\theta)\,\sigma^2} \leq 0.$$

7 General case

Here we generalize the model of Section 2 in two directions. First, we assume that $\pi = g(h,s)$ where $g : \mathbb{R}^2 \to \mathbb{R}$ is an increasing function of both arguments. Second, we assume that h and s are continuous random variables whose p.d.f. and c.d.f. are denoted by f_l and F_l for $l \in \{h, s, \pi\}$. Furthermore, we assume that f_l is positive over $\left[\underline{l}, \overline{l}\right]$ and, as before, we normalize its mean to zero. In order to establish the existence of the equilibrium, a series of definitions are required. First we define a number z as

$$\theta E\,(s - z | s > z)\Pr(s > z) = z \tag{16}$$

which, as in previous sections, will represent the price of soft information that prevails when h is certified. Also, we define $\hat{p}$ as

$$\theta E\,(\pi - \hat{p} | \pi > \hat{p})\Pr(\pi > \hat{p}) = \hat{p} \tag{17}$$

which will represent the maximum price that would prevail if certification was not available. The existence of both z and $\hat{p}$ was established by Marinovic (2010).

Now we can define a bound for the certification cost c. We will assume that g is such that there is a number $\overline{c} > 0$, defined by

$$\hat{p} = g\left(\overline{h}, z\right) - \overline{c}. \tag{18}$$

Setting $c < \overline{c}$ ensures that a positive probability of certification is feasible in equilibrium.

Finally, we define a set $S_0(x)$ as follows.

$$S_0(x) = \{(x,y) : \{g(x,y) \in [x, x+c]\} \cup \{g(x,y) > x+c, h < k(x)\}\} \tag{19}$$

where the function $k = k(x)$ is given by

$$x = g(k,z) - c. \tag{20}$$

The set $S_0(x)$ will represent the support of the manager's reporting strategy in the uncertified market when the maximum price is x. We are now ready to establish the existence of an equilibrium analogue to the one discussed in Section 2. Since, the basic structure is identical, we only discuss the existence of the maximum prices p, z and the certification threshold k.

Proposition 10 *There is an equilibrium characterized by three numbers, p, z and $k = k(p_0)$. Where p_0 and z solve*

$$\Delta_0(p_0,z) = \theta E(\pi - p_0 | (h,s) \in S_0(p_0)) \Pr((h,s) \in S_0(p_0)) + E(\pi - p_0 | h < k) F_h(k) = 0. \tag{21}$$

$$\theta E(s - z | s > z) \Pr(s > z) = z$$

and

$$p_0 = g(k,z) - c$$

Proof. The existence of z was proved by Marinovic (2010), so in the sequel we focus on proving the existence of p_0. First note that $\Delta_0(\cdot, z)$ is continuous all over $[\max(p_-, \underline{\pi}), p^+]$, where p_- is given by

$$p_- = g(\underline{h}, z) - c,$$

and

$$p^+ = g\left(\overline{h}, z\right) - c.$$

There are two cases. First, consider the case where $p_- > \underline{\pi}$. Observe that $\Delta_0(x,z)$ is proportional to the rents the market obtains from paying x to an uncertified firm claiming to have a value equal to or greater than x when x is the maximum price paid in that market. Of course, given the competitive nature of the market, an equilibrium exists if there is p satisfying Eq. (21). Now, it is easy to see that $\Delta_0(p_-, z) \geq 0$. In fact, observe that if p_- was the maximum price paid in the uncertified market, then only under $\tau = 1$

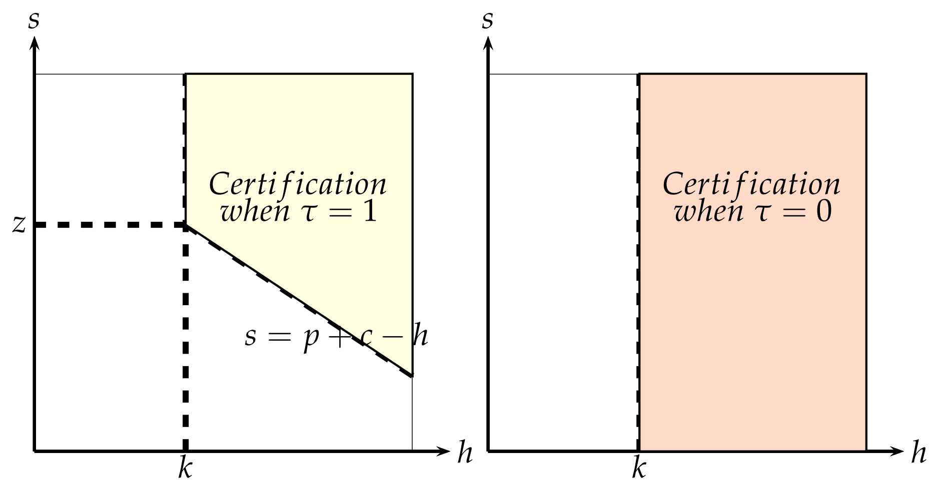

Figure 2: Equilibrium certification

the manager would both claim a value greater than p_- and choose not to certify h. Thus the market would necessarily make positive profits at that price. By contrast, p^+ is the price that would essentially shut down certification given certification cost c. On the other hand

$$\Delta_0 \left(p^+, z\right) = \Gamma \left(p^+, \theta\right)$$

where

$$\Gamma \left(p^+\right) = \theta E \left(\pi - p^+ | \pi > p^+\right) \Pr \left(\pi > p^+\right) - p^+.$$

But $\Gamma(\cdot)$ is a decreasing function. Furthermore, by the definition of $\hat{p}$, we know that $\Gamma(\hat{p}) = 0$. Since $c < \bar{c}$, then it is clear that $p^+ \geq \hat{p}$. Hence $\Delta_0 \left(p^+, z\right) \leq 0$. Therefore, by the Intermediate Value Theorem, there must be a $p_0 \in [p_-, p^+]$ such that $\Delta_0 \left(p_0, z\right) = 0$.
■

Figure 2 shows the structure of certification choices when one assumes that $g(h,s) = h + s$. There it becomes apparent that the probability of certification is lower under $\tau = 1$. We also see that certified firms are on average high value firms, yet, some uncertified firms are more valuable than some certified firms. For example, firms with very favorable soft information vis-a-vis their hard information sometimes are not certified when $\tau = 1$. By contrast, firms with very unfavorable soft information relative to their hard information may be certified under discretion, $\tau = 0$. It is also apparent that the propensity to certify information is greater under $\tau = 0$ than under $\tau = 1$.

8 Concluding remarks

A large portion of what we know about voluntary disclosure arises from two types of models: models in which the disclosure, when it occurs, is truthful but potentially costly, and models in which disclosures may be manipulated. In this paper, we bridge the gap between the two disclosure forms and develop a theory of choice over voluntary disclosure alternatives. That is, our model not only speaks about what information is disclosed but also how the information is disclosed.

The model provides a variety of new implications, many of which have not been, to our knowledge, empirically tested. Specifically, we predict that: (i) conditional on certification, managers make more aggressive reports about all other assets that may not be certified, (ii) investors discount these reports less than they would have absent certification, (iii) the likelihood of a fraud is greater conditional on a certification and (iv) negatively related to the size of the fraud, (v) the likelihood of frauds in the uncertified market is decreasing in the variance of hard assets and increasing in the certification costs, (vi) the size of overstatements is increasing in the volatility of the soft information and negatively related to the likelihood of frauds, (vii) managers with frauds are more likely to have certified information, (viii) certification is less likely and frauds are larger in markets with more perceived managerial credibility.

Lastly, we point to some of the inherent limitations of our model and to (what we believe) seem interesting avenues for further research in our context. First, our approach focuses on a single period and, as any such model, is subject to the very real caveat that a forward-looking manager anticipating the consequences on any leaked information (such as propensity to be untruthful) on future periods would not behave differently. In particular, by choosing to disclose or certify in a certain manner, the manager may acquire ex-post credibility and, thus, in future periods, achieve higher market prices. Extending the model to a dynamic setting would allow us to understand how managers build reputations over time. Second, we have focused on an environment in which hard and soft information are additively separable, leaving aside questions relating to hard and soft information being complements or substitutes. This excludes reasonable situations in which the firm holds a receivable and the value of that receivable is the product of the probability of payment with the size of the receivable (complements) or when a firm with great innovation capabilities can achieve high total value regardless whether some assets are already in place (substitutes). Third, since new investors are entirely price-protected (and make zero net surplus), we are unable to make any statements about the desirability of disclosure forms to capital providers, in particular relative to the interest of managers.

References

Bagnoli, Mark and Ted Bergstrom, "Log-Concave Probability and Its Applications," *Economic Theory*, 2005, *26* (2), pp. 445–469.

Benabou, Roland and Guy Laroque, "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility," *The Quarterly Journal of Economics*, August 1992, *107* (3), 921–58.

Botosan, Christine A., "Disclosure Level and the Cost of Equity Capital," *The Accounting Review*, 1997, *72* (3), pp. 323–349.

Brandenburger, Adam and Ben Polak, "When Managers Cover Their Posteriors: Making the Decisions the Market Wants to See," *RAND Journal of Economics*, Autumn 1996, *27* (3), 523–541.

Burdett, Kenneth, "Truncated means and variances," *Economics Letters*, 1996, *52* (3), 263 – 267.

Crawford, Vincent P. and Joel Sobel, "Strategic Information Transmission," *Econometrica*, 1982, *50* (6), pp. 1431–1451.

Dye, Ronald A., "Disclosure of Nonproprietary Information," *Journal of Accounting Research*, 1985, *23* (1), pp. 123–145.

Geanakoplos, John, David Pearce, and Ennio Stacchetti, "Psychological games and sequential rationality," *Games and Economic Behavior*, 1989, *1* (1), 60 – 79.

Gigler, Frank, "Self-Enforcing Voluntary Disclosures," *Journal of Accounting Research*, 1994, *32* (2), pp. 224–240.

Grossman, Sanford J., "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics*, 1981, *24* (3), pp. 461–483.

Heckman, James J. and Bo E. Honor, "The Empirical Content of the Roy Model," *Econometrica*, 1990, *58* (5), pp. 1121–1149.

Jovanovic, Boyan, "Truthful Disclosure of Information," *The Bell Journal of Economics*, 1982, *13* (1), pp. 36–44.

Kartik, Navin, Marco Ottaviani, and Francesco Squintani, "Credulity, lies, and costly talk," *Journal of Economic Theory*, 2007, *134* (1), 93 – 116.

Kreps, David M. and Robert Wilson, "Sequential Equilibria," *Econometrica*, 1982, *50* (4), pp. 863–894.

Lang, Mark and Russell Lundholm, "Cross-Sectional Determinants of Analyst Ratings of Corporate Disclosures," *Journal of Accounting Research*, 1993, *31* (2), pp. 246–271.

Levin, Jonathan, "Information and the Market for Lemons," *The RAND Journal of Economics*, 2001, *32* (4), pp. 657–666.

Lizzeri, Alessandro, "Information Revelation and Certification Intermediaries," *The RAND Journal of Economics*, 1999, *30* (2), pp. 214–231.

Milgrom, Paul R., "Good News and Bad News: Representation Theorems and Applications," *The Bell Journal of Economics*, 1981, *12* (2), pp. 380–391.

Morris, Stephen, "Political Correctness," *The Journal of Political Economy*, 2001, *109* (2), 231–265.

Ottaviani, Marco and Peter Norman Sorensen, "Reputational Cheap Talk," *The RAND Journal of Economics*, 2006, *37* (1), 155–175.

_ **and Peter Norman Srensen**, "The strategy of professional forecasting," *Journal of Financial Economics*, 2006, *81* (2), 441 – 466.

Pastena, Victor and Joshua Ronen, "Some Hypotheses on the Pattern of Management's Informal Disclosures," *Journal of Accounting Research*, 1979, *17* (2), pp. 550–564.

Selten, R., "Reexamination of the perfectness concept for equilibrium points in extensive games," *International Journal of Game Theory*, 1975, *4*, 25–55. 10.1007/BF01766400.

Sobel, Joel, "A Theory of Credibility," *The Review of Economic Studies*, 1985, *52* (4), 557–573.

Stocken, Phillip C., "Credibility of Voluntary Disclosure," *The RAND Journal of Economics*, 2000, *31* (2), 359–374.

Verrecchia, Robert E., "Discretionary disclosure," *Journal of Accounting and Economics*, 1983, *5*, 179 – 194.

A Omitted Proofs

Proof of Proposition 5: We decompose the proof into several steps.

Step 1. Show that $p_0 < \sigma + q$.

If $p_0 = \sigma + q$, then p_0 can only be attained for $r_0 = \sigma + q$ and no certification.[5] Then, all untruthful managers would report $\sigma + q$ and not certify, and thus $\mathbb{E}(h + s | r_0 = \sigma + q) = 0$, a contradiction.

Step 2. Show that $p_0 = P(\{d = 0, \sigma + q\})$.

Suppose by contradiction that the price p conditional on $r_0 = \sigma + q$ is strictly less than p_0. Consider now a sequence of perturbed games in which the manager must certify with probability in $[\epsilon/n, 1 - \epsilon/n]$, and with price conditional on $r_0 = \sigma + q$ defined by p_n. By (R), there exists n so that for any $n' \geq n$, $p_n < p_0$. This implies that, in the perturbed game, the untruthful manager would never report $\sigma + q$ but, then, $p_n = \sigma + q$. The latter is a contradiction to $p_0 < \sigma + q$.

Step 3. Show that $P(\{d = 0, r_0\}) = min(p_0, r_0)$. Note that this must be true for any equilibrium report, since $r_0 < p_0$ indicates that the report has been issued by a truthful manager. Consider an off-equilibrium report r and assume that it yields a price $p < r_0 \leq p_0$. There exists a sequence of perturbed games such that $P_n(\{d = 0, r\})$ converges to p and, therefore, for n large enough, $P_n(\{d = 0, r\}) < p_0$. This then implies that the manager must be truthful with probability one and $P_n(\{d = 0, r\}) = r$.

Step 4. Show that $P(\{d = 0, r_0\}) = h - c + min(z, s)$.

Consider a sequence of perturbed games where $P_n(\{d = 1, h, q\})$ converges to $P(\{d = 1, h, q\})$. Note that $P_n(\{d = 1, h, q\}) > 0$ since the reports may only have been made by a truthful with $s = q$ or an untruthful. It follows that the untruthful will necessarily report $r = q$ conditional on certifying h. Therefore, $P_n(\{d = 1, h, q\})$ can be obtained by Bayesian updating as:

$$P_n(\{d = 1, h, q\}) = \frac{\epsilon/2(\theta/2)}{\epsilon/2(1 - \theta + \theta/2)}$$

To conclude, note that conditional on reporting $s = -q$, the manager must be truthful with probability one.□

Proof of Proposition 6: The existence of a certification threshold follows from the monotonicity of the price function (R). Suppose h is certified by a manager with $(\tau, s) = (1, -q)$. Then, it must be that $h - q \geq p_0$ and $\min(p(h), h - q - c) \geq p_0$. Since the un-

[5]Recall that truthful managers could make the report but would need to have exactly $h + s = \sigma + q$ which is an event with probability zero.

truthful manager would never issue $r_1 = -q$ in this case, we can rewrite this condition as $h - q - c \geq p_0$. Consider a manager with $(\tau, s) = (1, q)$, then $h + q \geq p_0$, so that when not certifying the manager receives p_0, and $p(h) \geq h - c$ conditional on certifying. Therefore, the manager would certify. The certification condition for the truthful manager with $s = q$ is then given by $p_0 \leq p(h)$, which is the same condition as for the untruthful manager.□

B The meaning of A.5

We focus the analysis on situations where soft information is important. To formally define "important" we introduce the following notation. Define the number $w(c)$ by

$$\int_{-1}^{w} \frac{F(t)\,dt}{F(w)} = c. \tag{22}$$

and $k = k(\sigma, c)$ by

$$k = \sigma w\left(\frac{c}{\sigma}\right) \tag{23}$$

As we shall see, k pin-down the manager's certification threshold H_q. Before studying the properties of k, we introduce a number $\overline{c}$ defined as

$$\overline{c} = \int_{-\sigma}^{\sigma} F_h(t)\,dt.$$

which will represent the value of c that triggers the shut down of certification, so that

$$w\left(\frac{\overline{c}}{\sigma}\right) = 1.$$

Lemma 11 $\frac{\partial k}{\partial \sigma} \leq 0$ *and* $\frac{\partial k}{\partial c} \geq 1$. $\lim_{c\to 0} k = -\sigma$. $\lim_{c \to \overline{c}} k = \sigma$.

Proof. To show that $\frac{\partial k}{\partial c} \geq 1$, we apply the Implicit Function Theorem to (22), which gives

$$\frac{\partial w}{\partial c} = \frac{1}{1 - \frac{\partial}{\partial w}\int_{-1}^{w} \frac{F(t)dt}{F(w)}}.$$

where $\lambda(w) = \frac{f(w)}{F(w)}$. Now, the log-concavity of f_h implies that F_h and $\int_{-1}^{w} F(t)\,dt$ are log-concave too. Thus,

$$\frac{\partial^2}{\partial w^2} \log\left(\int_{-1}^{w} F(t)\,dt\right) = \frac{\partial}{\partial w} \frac{F(w)}{\int_{-1}^{w} F(t)\,dt} \leq 0$$

hence $\frac{\partial}{\partial w}\int_{-1}^{w}\frac{F(t)dt}{F(w)} \geq 0$, which in turn means that

$$\frac{\partial w}{\partial c} = \frac{1}{1 - \frac{\partial}{\partial w}\int_{-1}^{w}\frac{F(t)dt}{F(w)}} \geq 1 \Rightarrow \frac{\partial k}{\partial \sigma} \geq 1.$$

As for $\frac{\partial k}{\partial \sigma} \leq 0$, note that

$$\frac{\partial k}{\partial \sigma} = -\frac{\partial k}{\partial c}\frac{c}{\sigma^2} \leq 0.$$

■

We can now specify **A.5** as follows

$$q \geq \max\left(\frac{(\theta+2)(\sigma - c + k)}{2}, \frac{(\theta+2)(\sigma + c - k)}{2(\theta+1)}\right). \tag{24}$$

The main role of A.5 is to rule out the case in which an truthful manager certifies h even when soft information is bad, i.e., $s = -q$. That is, under **A.5**,

$$H_{-q} = \sigma.$$

C Proof of Proposition 8

Proof. To obtain z note that since $r_1 = q$, then z must satisfy

$$z = \frac{\gamma\frac{1}{2}q}{\gamma\frac{1}{2} + (1-\gamma)}$$

which implies

$$z = q\frac{\theta}{\theta+2}.$$

On the other hand, the value of p_0 must solve

$$\frac{\Pr(\tau = 1 | d = 0, x_0 \geq p_0)\, E(\pi - p | d = 0, x_0 \geq p_0, \tau = 1)}{\Pr(\tau = 0 | d = 0, x_0 \geq p_0)\, E(p - \pi | d = 0, x_0 \geq p_0, \tau = 0)} = 1. \tag{25}$$

Clearly,

$$\Pr(\tau = 0 | d = 0, x_0 \geq p_0) = \frac{(1-\gamma) F_h(k)}{\gamma \Pr(d = 0, x_0 \geq p_0 | \tau = 1) + (1-\gamma) F_h(k)}, \tag{26}$$

where $k = p_0 + c - z$. Also,

$$E(\pi | d = 0, x_0 \geq p_0, \tau = 0) = \int_{-\sigma}^{\min(k,\sigma)} h \frac{f_h(h)}{F_h(k)} dh. \tag{27}$$

Also,

$$\Pr(d = 0, x_0 \geq p_0 | \tau = 1) = \gamma \left(\int_{\max(p-q,-\sigma)}^{\min(k,\sigma)} \frac{1}{2} f_h(h)\, dh + \int_{\min(p+q,\sigma)}^{\min(p+c+q,\sigma)} \frac{1}{2} f_h(h)\, dh \right). \tag{28}$$

The first term arises when $s = -q$ and the second term arises when $s = q$. Plugging these expressions into (25) yields

$$\theta \frac{1}{2} \int_{\max(p-q,-\sigma)}^{\min(k,\sigma)} (h + q - p) f_h(h)\, dh + \theta \frac{1}{2} \int_{\min(p+q,\sigma)}^{\min(p+c+q,\sigma)} (h + q - p) f_h(h)\, dh$$
$$+ \int_{-\sigma}^{\min(p+c-z,\sigma)} (h - p) f_h(h)\, dh = 0$$

For now, assume that $p - q < -\sigma/2$ and $p + q > \sigma/2$. Then p must solve the simpler equation

$$\theta \frac{1}{2} \int_{-\sigma}^{k} (h + q - p) f_h(h)\, dh + \int_{-\sigma}^{k} (h - p) f_h(h)\, dh = 0. \tag{29}$$

It is easy to see that this equation is solved by $k = -\sigma$. Also, after integrating by parts, one can see that boils down to

$$\frac{\int_{-\sigma}^{k} F_h(t)\, dt}{F_h(k)} = c.$$

So long as $c \leq \bar{c} = \int_{-\sigma}^{\sigma} F_h(t)\, dt$, this equation has a unique solution in k. Thus

$$p_0 = k - c + z.$$

Furthermore, under A.5, both $p_0 - q \leq -\sigma$ and $p_0 + q \geq \sigma$.

To show that p_0 is the unique robust equilibrium. Note that if the truthful manager makes certification mistakes with probability ε, then in this perturbed game, there would be a threshold $k^\varepsilon = p_0^\varepsilon + c - z^\varepsilon$ such that when $h \geq k^\varepsilon$, both the untruthful and the truthful with $s = q$ would try to certify h. Whenever, the manager certifies $h < k^\varepsilon$ investors would realize the manager is truthful and the price would be set as

$$p(h) = h + x_1 - c < p(k^\varepsilon)$$

Anytime, the manager (correctly) certifies $h \geq k^{\varepsilon}$ and reports $x_1 = q$, the price of soft information, z^{ε}, would be given by

$$z^{\varepsilon} = \frac{\theta(1-\varepsilon)}{2+\theta(1-\varepsilon)} z.$$

Since uncertified reports would take place all over the support of π, the untruthful manager would randomize over $[p_0^{\varepsilon}, \sigma + q]$ when not certifying h. The value of p_0^{ε} would therefore solve

$$\frac{\theta(1-\varepsilon)}{2}\int_{-\sigma}^{k^{\varepsilon}}\left(h+q-p_0^{\varepsilon}\right)f_h(h)\,dh + \frac{\theta\varepsilon}{2}\int_{k^{\varepsilon}}^{\sigma}\left(h+q-p_0^{\varepsilon}\right)f_h(h)\,dh + \int_{-\sigma}^{k^{\varepsilon}}\left(h-p_0^{\varepsilon}\right)f_h(h)\,dh = 0. \tag{30}$$

By the Intermediate Value Theorem one can show that for any $c < \bar{c}$, there exists at least one p_0^{ε} that solves p_0^{ε}. Now, even if $(z^{\varepsilon}, p_0^{\varepsilon})$ was not unique,

$$\lim_{\varepsilon \to 0}\left(z^{\varepsilon}, p_0^{\varepsilon}\right) = (z, p_0).$$

■

D Proof of Corollary 2

Proof. The probability of certification is given by

$$\begin{aligned}
\Pr(d=1) &= \Pr(d=1|\tau=0)(1-\gamma) + \Pr(d=1|\tau=1)\gamma \\
&= \left(\frac{1}{2}\frac{\theta}{1+\theta} + \frac{1}{1+\theta}\right) F_h\left(-H_q\right). \\
&= \frac{1}{2}\frac{\theta+2}{1+\theta} F_h\left(-H_q\right)
\end{aligned}$$

■

E Proof of Corollary 3

Proof. By Bayes' rule, the probability of misreporting in the certified market is

$$\begin{aligned}
\Pr(\tau=0|d=1) &= \frac{\Pr(d=1|\tau=0)\Pr(\tau=0)}{\Pr(d=1|\tau=0)\Pr(\tau=0)+\gamma\Pr(d=1|\tau=1)} \\
&= \frac{(1-\gamma)F_h\left(-H_q\right)}{(1-\gamma)F_h\left(-H_q\right)+\gamma\left(\frac{1}{2}F_h\left(-H_q\right)\right)} \\
&= \frac{1}{1+\frac{\theta}{2}}
\end{aligned}$$

In the uncertified market,

$$\begin{aligned}
\Pr(\tau=0|d=0) &= \frac{(1-\gamma)F_h\left(H_q\right)}{(1-\gamma)F_h\left(H_q\right)+\gamma\left(\frac{1}{2}F_h\left(H_q\right)+\frac{1}{2}\right)} \\
&= \frac{1}{1+\frac{\theta}{2}\left(1+\frac{1}{F_h\left(H_q\right)}\right)}
\end{aligned}$$

■

F Proof of Corollary 4

Proof. The mean overstatement in the uncertified market is given by

$$\begin{aligned}
F_0 &= E\left[r_0\right] - E\left[\pi | h < H_q\right] \\
&= \int_{-\sigma}^{k+q} r\varphi_0\left(r\right) dr - E\left[h | h < H_q\right] \\
&= \int_{-\sigma}^{k+q} r\varphi_0\left(r\right) dr - \left(k-c\right) \\
&= q + \int_{-\sigma}^{k} r\theta \frac{r+q-p_0}{z}\frac{1}{2}\frac{f_h\left(r\right)}{F_h\left(k\right)} dr - k + c \\
&= q + c - k + \int_{-\sigma}^{k} r\theta \frac{r+q-k+c-z}{z}\frac{1}{2}\frac{f_h\left(r\right)}{F_h\left(k\right)} dr \\
&= q + c - k + \frac{\theta+2}{2q}\int_{-\sigma}^{k} r \frac{\left(r+q-k+c-z\right)}{1}\frac{f_h\left(r\right)}{F_h\left(k\right)} dr \\
&= q + c - k + \frac{\theta+2}{2q}\left(\int_{-\sigma}^{k} r^2 \frac{f_h\left(r\right)}{F_h\left(k\right)} dr + \left(q-k+c-z\right)\left(k-c\right)\right) \\
&= q + c - k + \frac{\theta+2}{2q}\left(\int_{-\sigma}^{k} r^2 \frac{f_h\left(r\right)}{F_h\left(k\right)} dr - \left(k-c\right)^2\right) + \frac{\theta+2}{2q}\left(q-z\right)\left(k-c\right) \\
&= q + c - k + \frac{\theta+2}{2q}\left(\int_{-\sigma}^{k} r^2 \frac{f_h\left(r\right)}{F_h\left(k\right)} dr - \left(k-c\right)^2\right) - \left(k-c\right) \\
&= q + \frac{\theta+2}{2q} Var\left(h | h < k\right).
\end{aligned}$$

where it becomes apparent that F_0 increases in θ. Furthermore, log-concavity of f_h ensures that $Var\left(h|h<k\right)$ increases in k (see e.g. Heckman and Honore, 1990). Thus F_0 must increase in c. To obtain the effect of q, note that

$$\frac{\partial F_0}{\partial q} = 1 - \frac{\theta+2}{2q^2} Var\left(h | h < k\right).$$

which clearly increases in q. By A.5, $q \geq q = \frac{(\theta+2)(\sigma+m)}{2}$, where

$$m = E\left(h | h < k\right) = k - c.$$

Thus, evaluating $\frac{\partial F_0}{\partial q}$ at $q = \frac{(\theta+2)(\sigma+m)}{2}$yields

$$\frac{\partial F_0}{\partial q} = 1 - \frac{2}{\theta+2}\frac{Var\left(h|h<k\right)}{\left(\sigma+m\right)^2} > 0.$$

On the other hand, the mean overstatement in the certified market is

$$F_1 = q.$$

■

G Proof of Corollary 5

Proof. The uniqueness of c^* is implied by the log-concavity of $F_h(\cdot)$. Note that

$$\begin{aligned} \max_c \Pi &\equiv \max_k \frac{\theta/2+1}{1+\theta}\left[1-F_h(k)\right]\frac{\int_{-\sigma}^{k} F_h(t)\,dt}{F_h(k)} \\ &= \max_w \frac{\theta/2+1}{1+\theta}\sigma\left[1-F(w)\right]\frac{\int_{-1}^{w} F(t)\,dt}{F(w)} \end{aligned}$$

Since, by choosing c, the certifier indirectly determines the threshold k, one can think of the certifier as choosing k (or w) rather than c. Now the log-concavity of F_h implies that $\frac{\int_{-\sigma}^{k} F_h(t)dt}{F_h(k)}$ is also log-concave. On the other hand, $[1-F_h]$ must be log-convex, thus it is not clear whether $\log \Pi$ is concave in k. However, a corner solution can never be optimal. So we are left with the possibility of multiple interior solutions. Now, maximizing $\log \Pi$ yields the first order condition

$$\frac{F_h(k^*)}{\int_{-\sigma}^{k^*} F_h(t)\,dt} = \frac{f_h(k^*)}{1-F_h(k^*)}. \tag{31}$$

where $k^* = k(c^*, \sigma)$. The right hand side is increasing, since the hazard rate of log-concave distributions is increasing (see e.g., Bagnoli & Bergstrom, 2005). By contrast, the left hand side is decreasing, by the log-concavity of f_h (see Burdett, 1996). Thus there can only be one solution to Eq. (31). ■

H Proof of Corollary 7

Proof. The effect of c is implied by Corollary 5. Consider now the effect of σ. Assume now that c is endogenous, so that $c = c^*$. Then by the envelope theorem:

$$\begin{aligned} \frac{\partial EF}{\partial \sigma} &= \frac{\gamma}{2} \int_{\frac{K-q}{\sigma}}^{1} tf(t)\,dt - \left(\frac{\gamma}{2} + (1-\gamma)\right)(1 - F(w^*)) \frac{\int_{-1}^{w^*} F(t)\,dt}{F(w^*)} \\ &= \frac{\gamma}{2}\left(H\left(\frac{K-q}{\sigma}\right) - \Gamma(w^*)\right) - (1-\gamma)\Gamma(w^*) \end{aligned}$$

where $w^* = \frac{k^*}{\sigma}$,

$$L(w^*) = (1 - F(w^*)) \frac{\int_{-1}^{w^*} F(t)\,dt}{F(w^*)},$$

and

$$H\left(\frac{K-q}{\sigma}\right) = \int_{\frac{K-q}{\sigma}}^{1} tf(t)\,dt.$$

Now both $\Gamma(\cdot)$ and $H(\cdot)$ are single-peaked functions. It is easy to see that $H(\cdot)$ is maximized at zero. So $H\left(\frac{K-q}{\sigma}\right) \leq H(0)$. Moreover,

$$L(0) = \Gamma(0).$$

Finally, by revealed preferences we know that

$$L(r^*) \geq \Gamma(0) = H(0),$$

hence, when $c = c^*$,

$$\frac{\partial EF}{\partial \sigma} \leq 0.$$

■

I Outside A.5

Here we consider the case where

$$\begin{aligned} p_0 + q &> \sigma \\ p_0 - q &< -\sigma \end{aligned}$$

where p_0 solves the following equation

$$\Delta_0 = \frac{\theta}{2}\int_{p_0-q}^{k}(h+q-p_0)\,dF_h + \int_{-\sigma}^{k}(h-p_0)\,dF_h = 0 \tag{32}$$

or

$$cF_h(k) = \int_{-\sigma}^{k}F_h(t)\,dt - \frac{\theta}{\theta+2}\int_{-\sigma}^{p_0-q}F_h(t)\,dt \tag{33}$$

where

$$k = p_0 + c - z.$$

Corollary 8 *There is a unique p_0 and p_0 increases in θ, and c and q.*

Proof. To show uniqueness note that

$$\frac{\partial\Delta}{\partial p_0} = \frac{\theta+2}{2}\left(\lambda(k)\left(\int_{-\sigma}^{k}F_h(t)\,dt - \frac{\theta}{\theta+2}\int_{-\sigma}^{p-q}F_h(t)\,dt\right) - F_h(k)\right)$$

where $\lambda(k) = \frac{f_h(k)}{F_h(k)}$ is the inverse hazard rate of F_h. λ is decreasing because f_h is log-concave. Therefore

$$\begin{aligned}\frac{\partial\Delta}{\partial p_0} &\leq \frac{\theta+2}{2}\left(F_h(k) - \frac{\theta}{\theta+2}\lambda(k)\int_{-\sigma}^{p_0-q}F_h(t)\,dt - F_h(k)\right)\\ &= -\frac{\theta}{2}\lambda(k)\int_{-\sigma}^{p_0-q}F_h(t)\,dt < 0.\end{aligned}$$

To show that p_0 increases in θ, by the Implicit Function Theorem we just need to show that $\Delta_\theta \geq 0$.

$$\begin{aligned}\Delta_\theta &= \frac{1}{2}\int_{p-q}^{k}(h+q-p)\,dF_h + \frac{\theta}{2}(c-z+q)\,f_h(k)\,z' + (c-z)\,f_h(k)\,z'\\ &= \frac{1}{2}\int_{p-q}^{k}(h+q-p)\,dF_h + \left(\frac{\theta}{2}(c-z+q) + (c-z)\right)f_h(k)\,z'\\ &= \frac{1}{2}\int_{p-q}^{k}(h+q-p)\,dF_h + \frac{\theta+2}{2}cf_h(k)\,q\frac{2}{(\theta+2)^2}\\ &= \frac{1}{2}\int_{p-q}^{k}(h+q-p)\,dF_h + \frac{cf_h(k)\,q}{(\theta+2)} > 0\end{aligned}$$

To show that p_0 increases in c note that

$$\Delta = \frac{\theta}{2}\int_{p-q}^{k}(h+q-p)\,dF_h + \int_{-\sigma}^{k}(h-p)\,dF_h$$

Thus

$$\begin{aligned}\Delta_c &= \left(\frac{\theta}{2}(c-z+q)+(c-z)\right)f_h(k)\frac{\partial k}{\partial c} \\ &= \frac{\theta+2}{2}cf_h(k)>0.\end{aligned}$$

The case of q is obvious. ■

Corollary 9 *k increases in c, θ and q.*

Proof. To show this note that

$$\begin{aligned}\Delta &= \frac{\theta}{2}(c-z+q)F_h(k)+(c-z)F(k)-\frac{\theta}{2}\int_{p-q}^{k}F_h(t)\,dt-\int_{-\sigma}^{k}F_h(t)\,dt \\ &= \frac{\theta+2}{2}cF_h(k)-\frac{\theta}{2}\int_{p-q}^{k}F_h(t)\,dt-\int_{-\sigma}^{k}F_h(t)\,dt\end{aligned}$$

Then for fixed k,

$$\begin{aligned}\Delta_\theta &= \frac{1}{2}cF_h(k)-\frac{1}{2}\int_{p-q}^{k}F_h(t)\,dt \\ &\propto cF_h(k)-\int_{p-q}^{k}F_h(t)\,dt\end{aligned}$$

Using 33 one gets

$$\begin{aligned}\Delta_\theta &\propto \int_{-\sigma}^{k}F_h(t)\,dt-\frac{\theta}{\theta+2}\int_{-\sigma}^{p-q}F_h(t)\,dt-\int_{p-q}^{k}F_h(t)\,dt \\ &> \int_{-\sigma}^{k}F(t)\,dt-\int_{-\sigma}^{p-q}F(t)\,dt-\int_{p-q}^{k}F(t)\,dt \\ &= \int_{p-q}^{k}F(t)\,dt-\int_{p-q}^{k}F(t)\,dt \\ &= 0\end{aligned}$$

The effect of c is straightforward from 32. ■