Firm Volatility in Granular Networks*

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Abstract

We propose a network model of firm volatility in which the customers' growth rate shocks influence the growth rates of their suppliers, larger suppliers have more customers, and the strength of a customer-supplier link depends on the size of the customer firm. Even though all shocks are i.i.d., the network model produces firm-level volatility and size distribution dynamics that are consistent with the data. In the cross section, larger firms and firms with less concentrated customer networks display lower volatility. Over time, the volatilities of all firms co-move strongly, and their common factor is concentration of the economy-wide firm size distribution. Network effects are essential to explaining the joint evolution of the empirical firm size and firm volatility distributions.

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1 Introduction

Recent research has explored how the network structure and firm size distribution in an economy can influence aggregate volatility. Acemoglu et al. (2012) and Carvalho (2010) show that sparsity of inter-sector linkages inhibits diversification in an economy and raises aggregate volatility. Gabaix (2011) points out that "granularity," or extreme skewness in firm sizes, concentrates economic mass among a few very large firms, similarly stifling diversification and increasing aggregate volatility.

This research is silent about the impact of networks and size concentration on the volatility of the firm. The volatility of firm-level stock returns and cash flows varies greatly over time (Lee and Engle (1993)) and across firms (Black (1976), Christie (1982), and Davis et al. (2007)). Such fluctuations in uncertainty have important implications for investment and hiring decisions and firm value, as highlighted by Bloom (2009).[1] But the underlying determinants of firm volatility are poorly understood. In much of the work on volatility in economics and finance, firms are modeled to have heteroscedastic shocks without specifying the source of heteroscedasticity. Our goal is to understand, both theoretically and empirically, how inter-firm linkages and size distributions interact to endogenously produce heteroscedasticity at the firm level.

We propose a simple model in which firms are connected to other firms in a customer-supplier network. Firms' idiosyncratic growth rate shocks, which are homoscedastic, are transmitted in part to their trading partners. Differences in firms' network connections, and evolution of the network over time, impart total firm volatility with cross section and time series heteroscedasticity.

Our model has three assumptions. First, firms' growth rates are influenced by the growth rates of their customers. As a result, the firm-specific shocks propagate through the network via connected firms. Second, the probability of a customer-supplier link depends on the size of the supplier so that large firms typically supply to a higher number of customers. Third, the importance of a customer-supplier link depends on the size of the customer. Large customers have a stronger connection with their suppliers, presumably because they account

[1] See also Leahy and Whited (1996), Bloom, Bond, and Van Reenen (2007), Stokey (2012) and the papers cited therein.

for a large fraction of their suppliers' sales. We provide microeconomic evidence for all three assumptions based on the observed customer-supplier networks among Compustat firms.

In our model, firms are aggregators of their own idiosyncratic shocks and the shocks to connected firms. The sparsity and granularity of a firm's customer network, which in turn depend on the firm size distribution, determine how well a firm diversifies the shocks that it is exposed to and thus determine its volatility. By connecting the firm size distribution to network formation, our model generates a rich set of implications for volatility in the cross section and the time series, which we can test. We study data on firm-level sizes, volatilities, and customer-supplier linkages, establishing a new set of stylized facts about firm volatility and confirming the model's implications.

Firm-level volatilities exhibit a common factor structure where the factor is firm size dispersion in the economy. In the model, each supplier's network is a random draw from the entire population of firms, so that any firm's customer network inherits similar dispersion to that of the entire size distribution. An increase in dispersion slows down every firm's shock diversification and increases their volatility. In the data, we find that firm volatilities possess a strong factor structure, and we show that size dispersion explains 25% of the variation in (realized) firm volatilities, as much as is explained by average volatility, a natural benchmark.[2]

The factor structure implies strong time series correlations between moments of the size and volatility distributions. An increase in the size dispersion translates into higher average volatility among firms. It also raises the cross section dispersion in volatilities. In the time series, size dispersion has a 72% correlation with mean firm volatility and 79% with the dispersion of firm volatility. Our paper is the first to provide an economic explanation for the factor structure in firm-level volatility by connecting it to firm concentration. A persistent widening in the firm size dispersion should lead to a persistent rise in mean firm volatility. We observe such a widening (increase in firm concentration) between the early 1960s and the late 1990s, providing a new explanation for the trend in mean firm volatility studied by

[2]Similarly, Engle and Figlewski (2012) document a common factor in option-implied volatilities since 1996, and Barigozzi, Brownlees, Gallo, and Veredas (2010) and Veredas and Luciani (2012) examine the factor structure in realized volatilities of intra-daily returns since 2001. Bloom, Floetotto, Jaimovich, and Terry (2012) shows that firm-specific sales growth and productivity show cross-sectional dispersion that fluctuates with the macroeconomy.

Campbell et al. (2001).

In the cross-section, differences in volatility across firms arise from two sources: differences in the number of a customers and differences in customer size dispersion. First, large firms are less volatile than small firms because they are connected to more customers, which improves diversification regardless of the size profile of its customer base. This effect also appears in the volatility factor structure in our model. Smaller firms have larger exposures to the common volatility factor, implying that small firms have both higher levels of volatility and higher volatility of volatility. In the data, doubling a firm's size lowers its volatility by 32%, and small firms indeed have higher volatility factor exposures, as predicted by the model.

Second, holding the number of connections fixed, a supplier's customer network is less diversified if there is more dispersion in the size of its customers. Because customer size determines the strength of a link, severe customer size disparity means that shocks to the biggest customers exert an outsized influence on the supplier, which raises the supplier's volatility. Differences in customer size disparity arise from probabilistic network formation – some suppliers will link to a very large (or very small) customer by chance alone. The data indeed show a strong negative cross-sectional relationship between firm size and firm volatility and a strong positive correlation between a firm's "out-Herfindahl," our measure of concentration in a firm's customer network, and its volatility. The dependence of firm volatility on firm size and firm out-Herfindahl survives the inclusion of other determinants of volatility previously proposed in the empirical literature, including industry concentration and competition, R&D intensity, equity ownership composition, firm age, and cohort effects. Collectively, this evidence supports a network-based explanation of firm volatility.

A calibrated version of our model matches the data not only qualitatively but also quantitatively. We target moments and cross-moments of data on firm size, firm volatility, and inter-firm business linkages.[3] A benchmark calibration with strong network effects is able to account quantitatively for most cross-sectional and aggregate features of the size, volatility, and linkage distributions. But it overstates the degree of concentration in customer net-

[3]While we are able to analytically characterize volatility behavior in the large-firm limit, ascertaining certain other model features necessitates a numerical simulation due to the inherent non-linearity of the network and its dynamics (exit and entry of firms and persistence in connections). The calibration also allows us to confront truncation and selection issues we face in the data.

works and underestimates the cross-sectional dispersion in volatility. An extension of the model that allows for internal diversification, for example from operating multiple product lines, is able to match the observed dispersion in volatility and degree of customer network concentration, while continuing to match the other size and volatility moments. We show that to match the joint distribution of volatility and size, we need both internal and external (network) diversification in the model.

The model displays substantial movement in aggregate moments of size and volatility distributions, despite allowing for thousands of firms, because the granular network introduces strong feedback between the cross-sectional size distribution and firm-level volatility: even small shocks to size dispersion have large dynamic volatility effects. The data display similarly large fluctuations as in our model. Without network effects, our model reduces to a Gibrat model and implies that the cross-sectional size and volatility distributions are effectively constant over time.

We build upon a rich literature exploring the role of input-output network linkages for aggregate fluctuations, exemplified by Long and Plosser (1987), Carvalho (2010), Foerster, Sarte, and Watson (2011), Acemoglu et al. (2012), and Carvalho and Gabaix (2013). Our model ties the size distribution to network formation, and shows that network connectivity is crucial for describing not only aggregate volatility, but the entire distribution of firm-level volatility. We combine the network sparsity insights of Acemoglu et al. (2012) with Gabaix's (2011) notion of limited diversification through granularity. The granularity of the network is related to the idea that a few large firms account for a large part of aggregate output and hence of aggregate volatility. Our granularity, by operating at the firm level, endogenously generates firm-level volatilities rather than taking those as given. While consistent with the economic structure in these papers, our model differs from earlier work by directly specifying the statistical mechanics of inter-firm linkages. This approach allows us to derive tractable analytic statements about firm volatility in large economies while still allowing for rich size and network dynamics.

The rest of the paper is organized as follows. Section 2 sets up the network model and studies its volatility implications. It also justifies the modeling assumptions based on micro data on firms' networks. Section 3 presents macro evidence on the link between the firm size

and the firm volatility distributions. Section 4 calibrates the model and studies its ability to match the size, volatility, and network data. The proofs of the theoretical results, auxiliary empirical evidence, and additional calibration results are relegated to the appendix.

2 A Network Model of Firm Growth and Volatility

We develop a dynamic network model of connections between firms. While the model is more general, our empirical application leads us to interpret the network as one between suppliers and customers.

2.1 Firm Growth

Define $S_{i,t}$ as the size of firm i with growth rate as $g_{i,t+1}$, where

$$S_{i,t+1} = S_{i,t} \exp(g_{i,t+1}). \tag{1}$$

In this directed network, supplier i's growth rates depend on its own idiosyncratic shock and a weighted average of the growth rates of its customers j:

$$g_{i,t+1} = \mu_g + \gamma \sum_{j=1}^{N} w_{i,j,t}\, g_{j,t+1} + \varepsilon_{i,t+1}. \tag{2}$$

The parameter $\gamma \in [0, 1)$ governs the rate of decay as a shock propagates through the network. The weight $w_{i,j,t}$ determines how strongly firm i's growth rate is influenced by the growth rate of firm j. If i and j are not connected then $w_{i,j,t} = 0$. By convention, we set $w_{i,i,t} = 0$. The full matrix of connection weights is $\boldsymbol{W}_t = [w_{i,j,t}]$. We assume that all rows of $\boldsymbol{W}_t$ sum to one so that its largest eigenvalue equals one. Connections are not symmetric: firm j can be a customer of i without i being a customer of j.[4] Let $\boldsymbol{g}_{t+1}$ and $\boldsymbol{\varepsilon}_{t+1} \sim N\left(0, \sigma_\varepsilon^2 \boldsymbol{I}\right)$

[4] As an aside, these growth dynamics impute dynamics to the relative sizes, or shares, of firms that are similar to those explored by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). In contrast to their work, there is no mean reversion built into our shares. Furthermore, only the customer shares in $\boldsymbol{W}_t$ are relevant for the cash flow dynamics of a firm. These shares divide by the sum of all customers not by the sum of all firms in the economy.

be the $N \times 1$ vectors of growth rates and shocks, respectively, so that:

$$\boldsymbol{g}_{t+1} = \boldsymbol{\mu}_g + \gamma \boldsymbol{W}_t \boldsymbol{g}_{t+1} + \boldsymbol{\varepsilon}_{t+1} = (\boldsymbol{I} - \gamma \boldsymbol{W}_t)^{-1} \left(\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}\right). \tag{3}$$

We purposely impose stark assumptions on the nature of the underlying innovations: each firm i experiences i.i.d. growth rate shocks $\varepsilon_{i,t+1} \sim N(0, \sigma_\varepsilon)$. Under this assumption, all dynamics in the volatility of growth rates arises endogenously.[5] In Section 4 we consider a model extension that allows for heterogeneity in firms' shock volatilities and find the same qualitative effects of network structure on firm volatility.

Acemoglu et al. (2012) derive a static version of (3) as the equilibrium outcome in a multi-sector production economy.[6] Theirs is also a directed network, but in their version the productivity shocks are transferred downstream from suppliers to customers.[7] The appendix explores a version of our model where shocks are propagated downstream from suppliers to customers. In Section 4, we argue that the data are more supportive of an upstream than a downstream shock propagation mechanism.[8]

2.2 Size Effects in Network Structure

So far, the statistical model for firm growth is a general representation of a spatial autoregression. The content of the network (spatial autoregression) model is in the specification of

[5] As in a Gibrat model, the model described here is non-stationary. Our theoretical analysis of firm volatility is unaffected by this property. In the calibration of Section 4, which includes time series moments, we introduce firm entry and exit into the model to maintain stationarity. This is a standard mechanism used in models of the size distribution (de Wit (2005)).

[6] Theirs is a constant-returns to scale economy populated by a stand-in agent who has Cobb-Douglas preferences defined over all of the N different commodities produced. Productivity shocks are transferred downstream from suppliers to customers. In this economy, the weights $w_{k,j}$, are the input weights of good k for firm j in the production of commodity j while γ measures the intermediate goods share. This specification emphasizes upstream supply shocks as the only drivers of firm-level volatility, while our model focuses on downstream demand shocks as the driver.

[7] Since we focus on propagation from customer to supplier, a question arises about retail firms whose customers are households. Because households are not part of the model, this would appear to be a source of leakage. In the data, we also consider retailers who do not have any customer connections with other firms, at least not in the standard sense. Our simple model can be extended to include retailers. If markets are incomplete, then some of the labor income risk that is specific to firms would show up in the consumption decisions of workers at these firms, which in turn would expose retail firms to the upstream risk.

[8] Shock transmission in the economy is plausibly bi-directional. We focus on uni-directional transmission to clearly highlight the network and size mechanisms of interest. Our setup is easily generalized, but at the cost of analytic tractability.

the weighting matrix $\boldsymbol{W}$. We make two key assumptions on the probability and the strength of connections between customers and suppliers. These assumptions connect the firm size distribution to the network structure of the economy. We provide micro-economic evidence for these assumptions in Section 2.6.

The linkage structure at time t is determined by the firm size distribution coming into period t. The existence of a link between i and j is captured by

$$b_{i,j,t} = \begin{cases} 1 & \text{if } i \text{ connected to } j \text{ at time } t \\ 0 & \text{otherwise,} \end{cases}$$

Each element of the connections matrix, $\boldsymbol{B}_t = [b_{i,j,t}]$, is drawn from a Bernoulli distribution with $P(b_{i,j,t} = 1)$. This connection probability is assumed to be linear in supplier size:

$$P(b_{i,j,t} = 1) \equiv p_{i,t} = \frac{\tilde{S}_{i,t}}{Z} N^{-\zeta} \text{ (for } i \neq j), \tag{4}$$

where $\tilde{S}_{i,t} = S_{i,t}/E[S_{i,t}]$ is the relative size of firm i versus the population mean and Z is a scale constant. While the precise functional form matters quantitatively, the crucial qualitative assumption is that the probability of a connection depends on the (relative) size of the supplier, not on the size of the customer. It follows immediately that larger suppliers have more connections on average. This is the model's first size effect.[9]

Equation (4) also builds sparsity into the network. The sparsity parameter $\zeta \in (0, 1)$ governs the speed at which the connection probability decreases in N. It implies that the the number of links in the system diverges as the number of firms goes to infinity, but that the probability of connecting to any single customer goes to zero. In a large economy, the expected number of customers (or out-degree) is:

$$N_{i,t}^{out} \approx N p_{i,t} = \frac{\tilde{S}_{i,t}}{Z} N^{1-\zeta}.$$

The number of linkages grows with the number of firms in the economy, but the rate of growth is slower when ζ is closer to 1.

[9]Empirical support of this assumption is discussed in Sections 2.6 and 4 below.

Conditional on a link existing between supplier i and customer j ($b_{i,j,t} = 1$), the strength of that link is linear in customer size:

$$w_{i,j,t} = \frac{b_{i,j,t} S_{j,t}}{\sum_{k=1}^{N} b_{i,k,t} S_{k,t}}, \quad \forall i, j, t. \tag{5}$$

This weighting scheme assumes that larger customers have a larger impact on a supplier's growth rate. This assumption is the second size effect in the network.

2.3 Firm Volatility

Conditional on $\boldsymbol{W_t}$, the variance-covariance matrix of growth rates $\boldsymbol{g}_{t+1}$ is given by:

$$\boldsymbol{V}_t \left(\boldsymbol{g}_{t+1} \right) = \sigma_\varepsilon^2 \left(\boldsymbol{I} - \gamma \boldsymbol{W}_t \right)^{-1} \left(\boldsymbol{I} - \gamma \boldsymbol{W}_t' \right)^{-1}. \tag{6}$$

The vector of firm volatilities is the square root of the diagonal of the variance-covariance matrix.

The so-called Leontief inverse matrix, $\left(\boldsymbol{I} - \gamma \boldsymbol{W}_t \right)^{-1}$, governs the behavior of firm volatility. In standard network settings, this inverse is an obstacle to deriving a tractable analytic characterization of volatility. Our model, in contrast, lends itself to a convenient volatility representation when the number of firms in the economy becomes large.

Before deriving our main result, it is useful to build intuition for volatility behavior in this network economy by considering a simplified version of the model. Suppose for a moment that growth rates follow the process

$$\boldsymbol{g}_{t+1} = \left(\boldsymbol{I} + \gamma \boldsymbol{W}_t \right) \left(\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1} \right). \tag{7}$$

In our full network model (2), a supplier's growth rate is influenced by each customer's growth rate, who is in turn influenced by their customers' growth rates, and so on, embedding a rich spatial autoregressive structure in firm growth. The simplification in (7) differs from the full network in that idiosyncratic shocks only propagate one step in the supply chain and then

die out. In fact, (7) is a first order approximation to (2), because

$$(\boldsymbol{I} - \gamma \boldsymbol{W}_t)^{-1} \quad = \quad \boldsymbol{I} + \gamma \boldsymbol{W}_t + \gamma^2 \boldsymbol{W}_t^2 + \gamma^3 \boldsymbol{W}_t^3 + \cdots \approx \boldsymbol{I} + \gamma \boldsymbol{W}_t,$$

under our maintained assumption that $\gamma \in [0, 1)$. In this system, the variance of firm's i growth rate simplifies to:

$$V_t \left(g_{i,t+1} \right) = V \left(\gamma \sum_j w_{i,j,t} \varepsilon_{j,t+1} + \varepsilon_{i,t+1} \right) = \sigma_\varepsilon^2 \left(1 + \gamma^2 H_{i,t} \right), \tag{8}$$

where

$$H_{i,t} \equiv \sum_{j=1}^{N} w_{i,j,t}^2$$

is the Herfindahl index of supplier i's network of customers. We refer to $H_{i,t}$ as the customer Herfindahl or the out-Herfindahl. This derivation shows that, to a first order approximation, the variance of a firm's growth rate is determined by its customer Herfindahl $H_{i,t}$, the volatility of the underlying innovations σ_ε^2, and the strength of shock transmission in the network γ.

The higher i's out-Herfindahl, the more concentrated is its customer network and the higher is its variance. A supplier's out-Herfindahl (and hence firm variance) is driven by two features, the number of customers it has and the amount of dispersion in its customers' sizes. The supplier is more diversified when it has more customers and when the dispersion in customer sizes is small. Because all firms, large and small, draw their connections from the same economy-wide size distribution, their customer networks have equal size dispersion in expectation. In our model, the key difference in the network structure across suppliers is the *number* of customers they have, which depends on the supplier's size through the Bernoulli probability function. This commonality in network concentration naturally leads to a factor structure in firm volatilities, where the factor is the dispersion of the economy-wide firm size distribution.

Returning to the full network specification in (2), the following results formalize the preceding intuition in a large N economy. First, we provide a limiting description of each supplier's customer Herfindahl. Throughout, we use asymptotic equivalence notation $x \sim y$

to denote that $x/y \to 1$ as $N \to \infty$. All proofs are in Appendix A. Where there is no ambiguity, we suppress time and/or firm subscripts.

Lemma 1. *Consider a sequence of economies indexed by the number of firms N. If S_i has finite variance, then*

$$H_i \sim \frac{1}{N^{(1-\zeta)}} \frac{Z}{\tilde{S}_i} \frac{E[S^2]}{E[S]^2}.$$

In particular, if $\log(S_i)$ is normal with variance σ_s^2, then

$$H_i \sim \frac{1}{N^{1-\zeta}} \frac{Z}{\tilde{S}_i} \exp(\sigma_s^2).$$

This lemma highlights the common structure in customer Herfindahls across suppliers. The ratio of the second non-central moment of the size distribution to the squared first moment captures the degree of concentration in the entire firm size distribution. Economy-wide firm size concentration affects the customer network concentration of all firms. Differences in customer Herfindahl across suppliers are inversely related to supplier size, capturing the model feature that larger firms are connected to more firms on average. Under lognormality, $E[S^2]/E[S]^2$ equals the (exponentiated) cross-sectional standard deviation of log firm size. The lemma applies more generally to the case where the size distribution has finite variance, and below we analyze firm volatility decay in the case of power law size distributions with infinite variance but finite mean.

As described above, H_i is the first order determinant of firm volatility in our model. The following intermediate lemma allows us to capture higher order network effects that contribute to a firm's variance.

Lemma 2. *Consider a sequence of economies indexed by the number of firms N. Define the matrix $\bar{W}$ as $[\bar{W}]_{i,j} = \tilde{S}_j/N$. If S_i has finite variance, then for $q = 2, 3, ...,$*

$$[W^q]_{i,j} \sim \frac{\tilde{S}_j}{N}.$$

Furthermore,

$$[W\bar{W}']_{i,j} \sim \frac{E[S^2]}{NE[S]^2} \quad \textit{and} \quad [\bar{W}\bar{W}'] \sim \frac{E[S^2]}{NE[S]^2}.$$

Our main proposition links the variance of a firm to its size and to the concentration of firm sizes throughout the economy.

Proposition 1. *Consider a sequence of economies indexed by the number of firms N. If S_i has finite variance, then the Leontief inverse has limiting behavior described by*

$$(I - \gamma W)^{-1} \sim I + \gamma W + \frac{\gamma^2}{1-\gamma}\bar{W},$$

and firm volatility has limiting behavior described by

$$V(g_i) \sim \sigma_\varepsilon^2 \left[1 + \left(\frac{\gamma^2}{N^{1-\zeta}}\frac{Z}{\tilde{S}_i} + \frac{2\gamma^3 - \gamma^4}{N(1-\gamma)^2}\right)\frac{E[S^2]}{E[S]^2} + \frac{2\gamma^2}{1-\gamma}\frac{\tilde{S}_i}{N}\right].$$

This proposition highlights the determinants of firm-level growth rate variance in a large economy. First, firm variance depends on economy-wide firm size dispersion $E[S^2]/E[S]^2$, familiar from Lemma 1. Overall size dispersion acts as a common factor across all volatilities, since more size dispersion makes every supplier less diversified, and this has ripple effects through the higher order terms (customers' customer networks are also less diversified, and so on). As γ approaches one, these higher order terms become quantitatively important.

Relative firm size, $\tilde{S}_i$, appears in two terms. Its primary role is in the first coefficient on $E[S^2]/E[S]^2$. Intuitively, larger firms have a lower exposure to the common size dispersion factor than do smaller firms because they typically connect to more customers, achieve better shock diversification, and display lower volatility. Smaller factor exposure also makes large firms less sensitive to fluctuations the firm size dispersion; that is, they display lower volatility of volatility.

The firm's relative size also appears in the numerator of the last term. Shocks to the largest firms are the most strongly propagated shocks in the model since these firms have the largest influence on their suppliers. These shocks feed back into large firms' own volatility while, in contrast, small firms' shocks die out relatively quickly in the network. Thus, the last term in Proposition 1 captures a countervailing increase in volatility for larger firms. In all of our numerical results, we find that the effect of the first size term dominates.

Finally, the variance of firm variance decays at rate $N^{1-\zeta}$, showing the dominance of H_i's

effect on total volatility. The dependence of volatility's decay rate on ζ captures the effect of network sparsity. High ζ means that there are relative few linkages compared to the size of the economy, which slows down the diversification of all firms.

Thus far we have assumed that the firm size distribution has finite variance, thus the slow volatility decay in Proposition 1 arises only due to network sparsity. Gabaix (2011) emphasizes that extreme right skewness of firm sizes can also slow down volatility decay in large economies. In the next result, we show that the firm-level network structure adds a mechanism to further slow down volatility decay beyond Gabaix's (2011) granularity mechanism, which depends on power law behavior of the size distribution.

Proposition 2. *Consider a sequence of economies indexed by the number of firms N. If S_i is distributed as a power law with exponent $\eta \in (1, 2]$, then firm variance decays at rate $N^{(1-\zeta)(2-2/\eta)}$.*

In the absence of network effects, power law sizes would imply that firm variance decays at rate $N^{2-2/\eta}$. For any given rate of decay determined by the power law, network sparsity further lowers the decay rate by ζ.

We have referred to the volatility structure described in Proposition 1 as a factor model. The next result formally characterizes volatilities comovement when N is large. Because H_i determines the rate of convergence for firm variance, we may understand how firm variances covary in a large economy by studying the asymptotic covariance among H_i and H_j.

Proposition 3. *Consider a sequence of economies indexed by the number of firms N. If S_i has finite fourth moment (e.g., if S_i is a lognormal variable), then the covariance between H_i and H_j has limiting behavior described by*

$$Cov(H_i, H_j) \sim \frac{1}{N^{1+2(1-\zeta)}} \frac{V(S^2)}{S_i S_j} \frac{Z^2}{E[S]^2}.$$

The covariance between $V(g_i)$ and $V(g_j)$ decays at the same rate.

As the number of firms grows, not only does the level of volatility decay, but so does its variance and covariance between the volatilities of different firms. Proposition 3 shows that

comovement among firm variances decays at rate $N^{1+2(1-\zeta)}$. Intuitively, covariance is lowest when both firms are large since large firms have low exposure to overall size concentration.

2.4 Aggregate Volatility

We next characterize the behavior of aggregate volatility. Let $\boldsymbol{S}_t$ be the vector of sizes, ι an N-vector of ones, and $\boldsymbol{\nu}_t = \boldsymbol{S}_t/\iota'\boldsymbol{S}_t$ the vector of size weights. We define the aggregate growth rate of the economy as:

$$g_{a,t+1} = \boldsymbol{\nu}_t'\boldsymbol{g}_{t+1} = \boldsymbol{\nu}_t' \left(\boldsymbol{I} - \gamma \boldsymbol{W}_t\right)^{-1} \left(\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}\right).$$

The variance of the aggregate growth rate is given by:

$$V_t(g_{a,t+1}) = \sigma_\varepsilon^2 \boldsymbol{\nu}_t' \left(\boldsymbol{I} - \gamma \boldsymbol{W}_t\right)^{-1} \left(\boldsymbol{I} - \gamma \boldsymbol{W}_t'\right)^{-1} \boldsymbol{\nu}_t.$$

A first observation is that variation in $\boldsymbol{W}_t$ and $\boldsymbol{S}_t$ induces heteroscedasticity in aggregate growth rates, even though all underlying innovations are i.i.d. across firms and over time. The next proposition shows that our network model drives a wedge between the rates of decay for aggregate versus firm-level variance.

Proposition 4. *Consider a sequence of economies indexed by the number of firms N. If S_i has finite variance, then firm volatility has limiting behavior described by*

$$V_t(g_{a,t+1}) \sim \frac{\sigma_\varepsilon^2}{(1-\gamma)^2} \frac{E[S^2]}{NE[S]^2}.$$

If S_i is distributed as a power law with exponent $\eta \in (1, 2]$, then aggregate variance decays at rate $N^{2-2/\eta}$.

When the variance of firm size is finite, aggregate variance decays at rate $1/N$, which is generally faster than the rate of decay of firm variance. That is, even if the firm size distribution is lognormal, there is slow volatility decay at the firm level but not at the aggregate level. For size distributions with infinite variance (but finite mean), aggregate

variance decays more slowly than $1/N$, but remains unaffected by ζ.[10]

2.5 Residual Volatility

In the literature, idiosyncratic volatility is typically constructed by first removing the aggregate component of growth rates with a statistical procedure such as principal component analysis, then calculating the volatilities of the residuals. In a granular network model like ours, such a factor regression approach is misspecified. There is no dimension-reducing common factor that fully captures growth rate comovement since, by virtue of the network, *every* firm's shock may be systematic. A sign of the misspecification of the factor model is that the residuals exhibit a volatility factor structure that looks very similar to the factor structure for total firm volatility. In our empirical analysis below we show that the key features of total volatilities also exist for volatilities of factor regression residuals.

2.6 Justifying Model Assumptions

Before we turn to the empirical relationship between the firm size and firm volatility distribution, we discuss how customer-supplier network micro-data support our main modeling assumptions.

Our data for annual firm-level linkages comes from Compustat. It includes the fraction of a firm's dollar sales to each of its major customers. Firms are required to supply customer information in accordance with Financial Accounting Standards Rule No. 131, in which a major customer is defined as any firm that is responsible for more than 10% of the reporting seller's revenue.[11] The Compustat data has been carefully linked to CRSP market equity data by Cohen and Frazzini (2008), which allows us to associate information on firms' network connectivity with their market equity value and return volatility.[12] The data set covers the

[10] As shown by Acemoglu et al. (2012), network sparsity may impact aggregate volatility. Our model, by focusing on size as the key determinant of network formation, combines the aggregate volatility effects of network sparsity and granularity into a single mechanism that acts through the power law parameter η.

[11] Firms have discretion in reporting relationships with customers that account for less than 10% of their sales, and this is occasionally observed. In our data, 23% of firms report at least one customer that accounts for less that 10% of its sales.

[12] Cohen and Frazzini (2008) used the same data to show that news about business partners does not immediately get reflected into stock prices. Atalay et al. (2011) also use Compustat sales linkage data to develop a model of customer-supplier networks.

period 1980-2009, and includes 48,839 customer-supplier-year observations. Appendix B.1 provides more details and summary statistics on the network data.

The first size assumption in the model is that suppliers have stronger connections with their larger customers. This is a prominent feature of the data. Specifically, for each supplier i in the Compustat data, we calculate the correlation between its customers' total sales ($S_{j,t}$) and the fraction of i's revenue that is due to each customer ($w_{i,j,t}$). We then average across all suppliers in a given year, and finally average across all years. We also compute a t-statistic equal to the ratio of the time series mean of the correlation to its time-series standard deviation. In an average year, that correlation is 20% ($t = 3.6$). The correlation is 19% when we limit ourselves to only use links that exceed 10% of sales.

Our second size assumption is that larger suppliers are connected to more customers on average (higher N_i). The Compustat data cannot speak directly to this assumption due to the truncation of all linkages whose weights fall below 10%.[13] To address this, we simulate a calibrated version of our model and estimate the correlation between supplier size and out-degree in both the full simulated sample and the simulated sample while imposing the 10% weight truncation. We find that truncation produces a downward bias in the correlation estimate of 19 to 27 percentage points depending on the calibration. Since the measured correlation in the actual (truncated) data is 3%, the bias-adjusted estimate is 22% to 30% based on our model. The data also support our assumption that the probability of a connection is independent of customer size (the truncation-adjusted correlation between customer size and number of suppliers is 4% in the data). We discuss these points further in Section 4.

Inter-industry trade data from the Bureau of Economic Analysis does not suffer truncation, so for comparison we test the correlation between size and out-degree at the industry level.[14] We find a 61% correlation between the size of a supplier industry and the number of industries that it sells to. We also find a 37% correlation between the size of the customer industry $S_{j,t}$ and importance of the link $w_{i,j,t}$.

[13] For example, a small firm that has a single customer accounting for 100% of its revenue can show up as having more links than a firm with 11 equally important customers with weights of 9.1% due to truncation.

[14] We use the BEA's input-output tables for the 65-industry breakdown of the US economy between 1998 and 2011.

3 Macro Evidence on Size Dispersion and Volatility

This section documents new stylized facts about the joint evolution of the firm size and firm volatility distributions.

3.1 Data

We conduct our analysis using stock market data from CRSP over the period 1926–2010 and using cash flow data from CRSP/Compustat over 1952–2010. We consider market and fundamental measures of firm size and firm volatility calculated at the annual frequency. For size, we use equity market value at the end of the calendar year or total sales within the calendar year.[15] Market volatility is defined as the standard deviation of daily stock returns during the calendar year. Fundamental volatility in year t is defined as the standard deviation of quarterly sales growth (over the same quarter the previous year) within calendar years t to $t+4$.[16]

In the data, size and volatility are well approximated by a lognormal distribution.[17] As a result, the dynamics of each distribution may be summarized with two moments: the cross section mean and standard deviation of the log quantities. We now examine the dynamics of these moments in detail.

3.2 Comovement of Size and Volatility Distributions

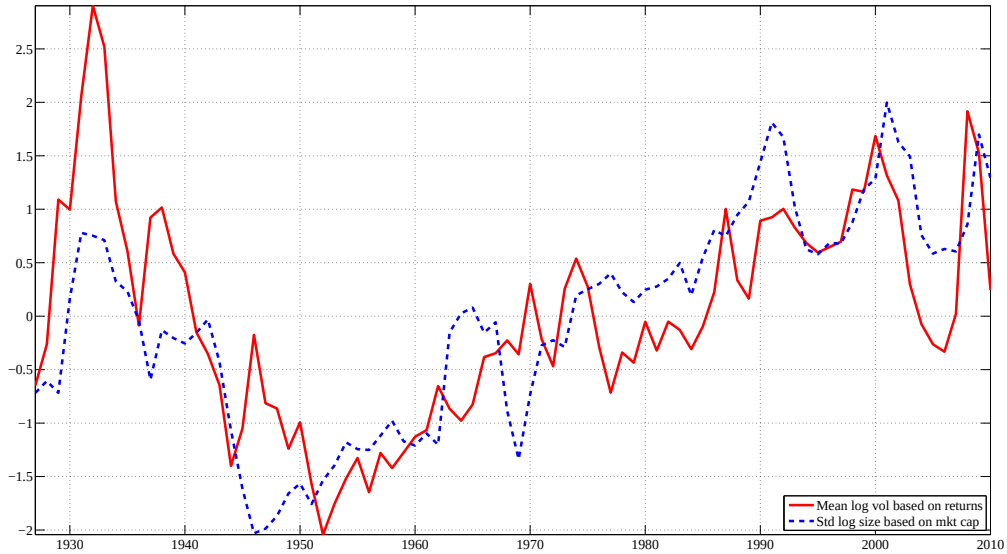
Figure 1 plots the cross-sectional *average* of log firm variance against lagged firm size dispersion, where size is based on market capitalization and volatility based on stock returns. The correlation between average volatility and the market-based measure of size dispersion is 71.7%. Mean firm volatility experienced several large swings in the past century, especially in the 1920s and 1930s and again in the last two decades of the sample. These changes are preceded by similar dynamics in the cross-sectional dispersion of firm size. The close

[15] All variables in our analysis are deflated by the consumer price index.

[16] We also consider fundamental volatility measured by the standard deviation of quarterly sales growth within a single calendar year. The one and five year fundamental volatility estimates are qualitatively identical, though the one year measure is noisier because it uses only four observations.

[17] Detailed distributional statistics for the CRSP/Compustat sample are available upon request.

Figure 1: Average Firm Volatility and Dispersion in Firm Size



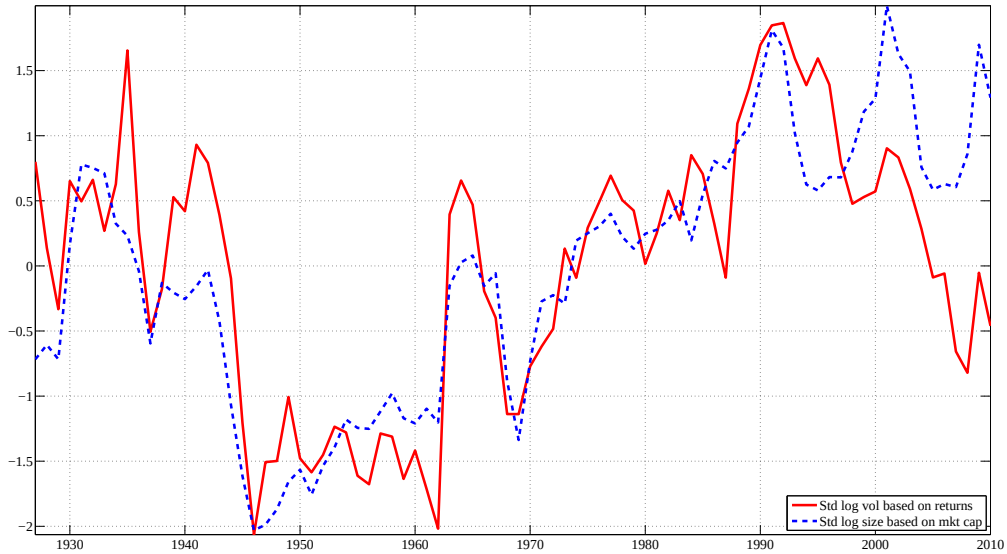
Notes: The figure plots the cross-sectional dispersion of log firm size, based on market equity values, and the cross-sectional mean of the log variance distribution, where the variance is measured based on daily stock returns. All series are rescaled for the figure to have mean zero and variance one.

association between mean volatility and size dispersion is predicted by Proposition 1.

Proposition 1 links not only the mean volatility, but also dispersion in firms' volatility, to dispersion of log firm size. Figure 2 shows a strong association between firm volatility and firm size dispersion, based on the market measures. The correlation between the two time-series is 79.3%. Appendix B.2 shows that the positive correlation between size dispersion and average volatility holds both at low and business cycle frequencies, based on HP filter trend and cycle components of these time series. Appendix B.3 reports formal Granger causality tests showing that size dispersion leads mean and dispersion of the volatility distribution. These tests are useful for establishing that the time-series connection is statistically large and not driven by trends in the data.

It is important to note that the same relationship between the moments of the firm size and firm volatility distributions exists for our sales-based measures. Figure 3 shows a correlation of 85.5% between mean volatility and lagged firm size dispersion. Because the sales-based data only start in the 1960s, their dynamics are more affected by the persistent increase in firm size dispersion and volatility that took place between the 1960s and the 1990s.

Figure 2: Dispersion in Volatility and Dispersion in Firm Size



Notes: The figure plots the cross-sectional dispersion of log firm size, measured based on market equity values, and the cross-sectional dispersion of the log variance distribution, where the variance is measured based on daily stock returns. All series are rescaled for the figure to have mean zero and variance one.

The correlation between the dispersion of volatility and the dispersion of size is 72.4% for the sales-based measure, consistent with the market-based evidence. Average return volatility and average sales growth volatility have an annual time series correlation of 64%, while the two volatility dispersion measures have a correlation of 49%.[18] This demonstrates a high degree of similarity between market volatilities and its (more coarsely measured) fundamental counterpart. Any explanation of these volatility facts, including financial explanations, must confront this similarity.

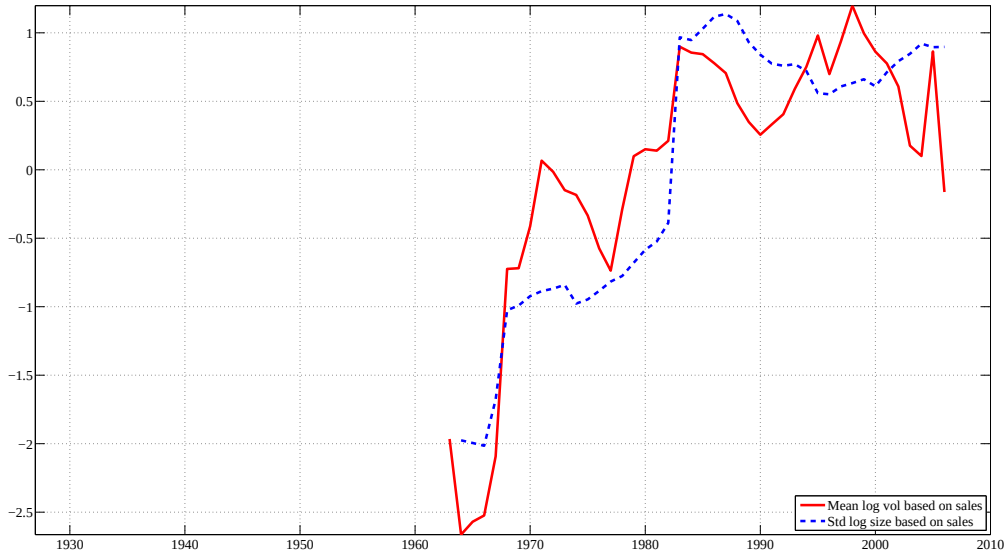
To summarize, we observe a strong positive association of firm size dispersion with average firm volatility and dispersion in firm volatility, as predicted by the model.

3.3 Sample Composition

Davis et al. (2007) show that, starting in the 1980s, firms go public earlier, at a stage in their life-cycle where they are more volatile. This fact does not appear to be driving the

[18] Using a different measure of volatility, (Bloom, Floetotto, Jaimovich, and Terry 2012) also find a strong positive correlation of 63% (annual) and 47% (quarterly) between stock return volatility and sales growth volatility of publicly listed firms.

Figure 3: Average Firm Volatility and Dispersion in Firm Size



Notes: The figure plots the cross-sectional dispersion of log firm size, based on firm sales, and the cross-sectional mean of the log variance distribution, where the variance is measured based on 20 quarters of growth in firm sales. All series are rescaled for the figure to have mean zero and variance one.

comovement between size and volatility shown in Section 3.2. High correlation between firm size dispersion and the moments of the volatility distribution holds both among young firms and firms that have been publicly listed for decades. It holds both before and after 1980. It holds within all industries and for the NASDAQ subsample (a large fraction of post-1980 IPOs were on NASDAQ and/or in the technology sector). Table 1 demonstrates that the positive correlations between firm size dispersion and both the mean and dispersion of firm variance are robust to sample composition. These correlations hold across 10 broad industries, across size terciles, for NYSE firms and non-NYSE firms, for firms that have been publicly listed at least 50 years and for random samples of 500 firms. Hence, these correlations unlikely to be driven by changes in the nature of publicly listed firms.

We also find that the dynamics of both firm size dispersion and firm volatility dispersion are similar for publicly-listed and privately-held firms. We use three data sets to corroborate this claim. First, using Census data, we document that the dispersion in the log size distribution, where size is measured by number of employees, displays similar dynamics for publicly listed and privately-held firms. The two series have a correlation of 65%. Second, using data

Table 1: Composition

	# Firms	$\rho(\sigma_{\text{subset},S,t}, \sigma_{S,t})$	$\rho(\mu_{\sigma,t}, \sigma_{S,t-1})$	$\rho(\sigma_{\sigma,t}, \sigma_{S,t-1})$
All stocks	3004	-	71.7%	79.3%
By sample period / exchange				
NYSE only	1158	64.2%	62.1%	77.6%
Non-NYSE	3018	89.9%	58.1%	40.7%
At least 50 yrs	347	78.1%	44.5%	62.7%
Random 500	500	90.5%	64.9%	80.7%
By size				
Smallest third	1000	67.7%	71.7%	51.9%
Middle third	1000	87.7%	61.6%	69.8%
Largest third	1003	86.8%	55.9%	73.4%
By industry				
Consumer Non-Dur.	248	91.4%	64.4%	72.3%
Consumer Durables	107	87.6%	33.4%	74.9%
Manufacturing	528	91.9%	53.1%	79.8%
Energy	140	72.9%	68.4%	67.9%
Technology	413	88.1%	85.2%	58.2%
Telecom	55	23.6%	14.3%	10.6%
Retail	310	86.5%	69.0%	69.8%
Healthcare	188	69.5%	69.4%	50.6%
Utilities	112	67.8%	19.7%	64.8%
Other	904	82.8%	63.3%	65.7%

Notes: Annual data 1926-2010. We use the market based volatility measure constructed from stock returns and the market-based measure of size (market equity). The first column reports the time series average of the cross section number of firms. The second column reports the time-series correlation between the size dispersion of the sub-cross section and the dispersion of the full cross section. Column three reports the correlation between average log volatility ($\mu_{\sigma,t}$) and lagged log size dispersion ($\sigma_{S,t-1}$) and column four reports the correlation between dispersion in log volatility ($\sigma_{\sigma,t}$) and lagged log size dispersion.

provided by Bloom et al. (2012), we find a strong positive correlation between stock return and sales growth volatility of publicly listed firms and the volatility of log productivity and of output growth for all public and private firms in manufacturing. Sales growth volatility (stock return volatility) has a 49% (33%) correlation with log TFP volatility 38% (52%) correlation with output growth volatility. Third, using data from Compustat on privately-held firms, we again find strong positive correlations between the time series of size dispersion, mean volatility, and volatility dispersion for public and for private firms. The correlations

range from 55% to 91%. Appendix B.4 provides data sources and detailed results.

Our results focus on data for publicly-listed firms because these are the firms for which we observe customer-supplier network relationships. The key stylized facts linking the size and volatility distributions hold broadly and are not an artefact of public firm selection.

3.4 Volatility Factor Structure

Recent research has documented a puzzling degree of common variation in the panel of firm-level volatilities. Kelly, Lustig, and Nieuwerburgh (2012) show that firm-level stock return volatilities share a single common factor that explains roughly 35% of the variation in log volatilities for the entire panel of CRSP stocks. This R^2 is nearly twice as high for the 100 Fama-French portfolios. They also show that this strong factor structure is not only a feature of return volatilities, but also of sales growth volatilities. The puzzling aspect of this result is that the factor structure remains nearly completely intact after removing all common variation in returns or sales growth rates. Hence, common volatility dynamics are unlikely to be driven by an omitted common return or sales growth factor.

The granular network model predicts a high degree of comovement in firm volatility. Proposition 1 shows an approximate factor structure among the volatilities of all firms, and suggests that concentration of the lagged economy-wide size distribution is the appropriate factor. Furthermore, as discussed in Section 2.5, if the true data generating process is a network model, then factor model residual volatilities will possess a similar degree of comovement as total volatilities, despite residual growth rates themselves being nearly uncorrelated.

Panel A of Table 2 shows results of panel volatility regressions for three different factor models. The left three columns use the volatility of total returns and total sales growth rates, while the right three columns use residual volatilities. Residual volatilities are calculated in a one factor model regression of stock returns (sales growth rates) on the value-weighted market return (sales-weighted average growth rate). In both cases, residuals have average pairwise correlations that are below 2% in absolute value, despite the original series having average correlations over 25% on average. Columns (1) and (4) consider the dispersion of lagged log market-based firm size as a factor. Columns (2) and (5) consider the lagged weighted-

average volatility. This lagged cross-sectional average volatility is a natural benchmark for factor model comparison because it is essentially a first principal component of volatilities and because the lag maintains a comparable timing with the conditioning factor implied by our model. The third column instead uses the contemporaneous average volatility as a factor. Because it uses finer conditioning information, it can be considered an upper bound on the explanatory power of a single factor. We report panel R^2 values based on each factor.

Lagged size dispersion has the same degree of explanatory power as lagged average volatility. Both factors capture about 25% of the panel variation in return volatility. The contemporaneous mean volatility explains closer to 40% of the variation. The results for sales-based volatility are similar: our size dispersion factor explains about 22% of the panel variation, close to the 23% and 24% for the lagged and contemporaneous average volatility factor. We find the same results using volatilities of factor model residuals.

The model also predicts that larger firms have lower loadings on the size dispersion factor, which lowers the level of their volatilities relative to small firms, and also lowers their time-series volatility of volatility. Panel B of Table 2 shows the loading of firm volatility on each factor, averaged within each quintile of the firm size distribution. Small firms have loadings that are 50% larger on the factor than large firms. Panel C shows that the common factor explains a larger share of the volatility dynamics of small firms than that of large firms. Large firms have lower levels of volatility and also less variation in volatility. Table 2 is consistent with the model's prediction that dispersion in firm sizes predicts the entire panel of firm-level volatilities.

3.5 Determinants of Firm-level Volatility

A large literature has examined the determinants of firm level volatility on the basis of firm characteristics, including Black (1976) who proposed that differences in leverage drive heterogeneity in firm volatility, Comin and Philippon (2006) who argue for industry competition and R&D intensity, Davis et al. (2007) who emphasize age effects, and Brandt et al. (2010) who argue that institutional ownership is a key driver of volatility. Our model predicts a negative correlation between volatility and firm size and a positive correlation between volatility and customer network concentration (out-Herfindahl).

Table 2: R^2 of Volatility Factor Models

	Total Volatility			Residual Volatility		
	(10	(2)	(3)	(4)	(5)	(6)
	Factors			Factors		
	$\sigma_{S,t-1}$	$\mu_{\sigma,t-1}$	$\mu_{\sigma,t}$	$\sigma_{S,t-1}$	$\mu_{\sigma,t-1}$	$\mu_{\sigma,t}$
Panel A: Factor Model R^2, All Firms						
Return Vol.	24.4	25.9	39.3	24.7	26.5	37.5
Sales Gr. Vol.	21.8	23.4	24.3	21.4	25.8	27.8
Panel B: Return Volatility Loadings by Size Quintile						
Q1	1.25	1.09	1.18	1.32	1.10	1.19
Q5	0.80	0.65	0.84	0.81	0.55	0.71
Panel C: Return Volatility R^2 by Size Quintile						
Q1	47.9	76.5	90.9	52.6	77.2	90.6
Q5	37.7	53.7	87.2	48.9	49.2	79.1

Notes: The table reports factor model estimates for the panel of firm-year volatility observations. In Panel A, total volatility is measured as standard deviation of daily returns within the calendar year, and residual volatility is estimated from daily regressions of firm returns on the value-weighted market portfolio within the calendar year. In Panel B, total volatility in year t is measured as standard deviation of quarterly observations of year-on-year sales growth for each stock in calendar years t to $t+4$. Residual volatility is measured from regressions of firms sales growth on the sales-weighted average growth rate for all firms. All volatility factor regressions take the form $\log \sigma_{i,t} = a_i + b_i \text{factor}_t + e_{i,t}$. We consider three different volatility factors. The first, motivated by our network model, is the lagged cross section standard deviation of log market equity, $\sigma_{S,t-1}$. The second and third factors we consider are the lagged and contemporaneous cross section average log volatility, $\mu_{\sigma,t-1}$ and $\mu_{\sigma,t}$. We report the pooled factor model R^2 in percent.

Table 3 reports panel regressions of firm-level log annual return volatility on size and out-Herfindahl, controlling for a range of firm characteristics including log age, leverage, industry concentration, institutional holdings, and industry and cohort fixed effects.[19] Consistent with our model, we find that the two most important determinants of volatility are size and out-Herfindahl. Doubling the size of a firm decreases its volatility by between 12% and 16%. An increase of customer Herfindahl from zero to one increases volatility by 88% without controlling for size; the effect is 17% when we control for size. Note that in our model size and network concentration convey similar information since size determines network structure. Within our network model, a supplier's size and its customer Herfindahl are

[19]Cohorts are defined by the year in which a firm first appears in the CRSP/Compustat data set.

strongly negatively correlated in the cross-section. Given that concentration in the sales network is measured with substantial noise, it is likely that size captures an important part of the true network concentration effect. Nevertheless, the table provides strong evidence that network concentration matters separately for firm volatility and survives the inclusion of other well-known volatility determinants such as firm age.

When we replace the out-Herfindahl (concentration of the customer network) with the in-Herfindahl (concentration of the supplier network) in this multivariate regression, network concentration is no longer a significant determinant of firm-level volatility (untabulated). This fact supports the upstream transmission of shocks we assume in our network model.

3.6 Rise in Firm-level Volatility

In an influential paper, Campbell et al. (2001) highlight the rise in firm-level volatility among publicly traded firms between the 1960s and 1990s. Indeed, our data confirm that the volatility of firm-level stock returns has increased from an average of 26% per year in the 1950s to 63% per year since 1990. This increase is also present in volatilities of residuals from a factor model for returns. The increase in total and residual firm volatility has puzzled financial economists, who have offered both "real" and "financial" explanations. Our stylized facts about the joint distribution of volatility and size, which manifest themselves similarly in both fundamental and market volatility measures, seem to point in the direction of a real explanation rather than a financial one.

Our model predicts that average volatility should trend upward if size dispersion is doing the same. We find that accounting for changes in the size distribution nullifies the volatility trends of the 1950-2000 sample highlighted by Campbell et al. (2001). A regression of mean firm variance on our market-based measure of size dispersion (the cross-sectional dispersion in log market equity) has an R^2 of 51.3%. There is no anomalous trend in average firm volatility in the post-war era after accounting for movements in the size distribution.

Table 3: Determinants of Firm-Level Volatility

	Dependent Variable: Log Firm Volatility														
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Log Sales	-0.16		-0.14	-0.15		-0.12	-0.14		-0.12	-0.13		-0.12	-0.13		-0.12
	-16.71		*-15.57*	*-25.56*		*-23.93*	*-19.32*		*-12.10*	*-33.10*		*-19.93*	*-20.61*		*-17.78*
H^{out}		0.88	0.17		0.63	0.17		0.53	0.15		0.55	0.13		0.74	0.17
		7.14	*3.09*		*8.74*	*3.43*		*7.01*	*2.65*		*8.69*	*3.27*		*28.54*	*8.81*
Log Age				-0.05	-0.26	-0.11	-0.08	-0.22	-0.11						
				-3.07	*-11.32*	*-6.60*	*-3.52*	*-6.58*	*-6.10*						
Leverage				0.29	0.10	0.24	0.30	0.04	0.23	0.32	0.13	0.26			
				3.32	*1.08*	*3.10*	*4.53*	*0.48*	*3.45*	*4.71*	*1.87*	*3.95*			
Ind. Conc.				0.32	0.36	0.35	0.44	0.41	0.37	0.32	0.30	0.29			
				1.51	*1.32*	*1.62*	*2.13*	*1.35*	*1.70*	*2.06*	*1.45*	*1.81*			
Inst. Hldg.							-0.03	-0.50	-0.06						
							-0.39	*-5.09*	*-0.61*						
Constant	-3.91	-3.52	-3.78	-3.92	-3.03	-3.64	-3.80	-2.93	-3.60	-4.01	-3.64	-3.88	-3.86	-3.51	-3.75
	-88.45	*-54.43*	*-76.93*	*-49.72*	*-39.79*	*-42.73*	*-32.42*	*-30.61*	*-30.98*	*-72.31*	*-50.66*	*-59.97*	*-111.89*	*-94.74*	*-82.52*
FE	None	None	None	None	None	None	None	None	None	Cohort	Cohort	Cohort	Ind.	Ind.	Ind.
Obs.	171,034	38,030	37,202	145,070	32,887	32,887	113,425	31,318	31,318	145,247	32,901	32,901	171,034	38,030	37,202
Adj. R^2	0.329	0.048	0.361	0.347	0.228	0.402	0.354	0.281	0.413	0.405	0.307	0.448	0.386	0.218	0.425

Notes: The table reports panel regressions of firm's return volatility on a range of characteristics including size (log sales), customer network Herfindahl (H^{out}), age, book leverage, competition measured by industry size concentration, ownership composition measures as fraction of shares held by 13f-reporting institutions, as well as cohort and industry fixed effects in certain specifications. Cohorts are defined by the year in which a firm first appears in the CRSP/Compustat data set. Industries are defined according based on 4-digit GICS codes. Standard errors are clustered at the industry level.

4 Joint Dynamics of Size and Volatility Distributions

Section 3 documents a strong statistical association between the distributions of size and volatility that matches the qualitative predictions of our model. In this section we ask whether our model can provide a successful quantitative description of the joint distribution of data on firm size, firm volatility, and inter-firm network linkages. In addition, the calibration allows us to address certain limitations that we face in our data, such as biases arising from public firm selection or truncation of weights below 10%. It also allows us to add further realism to the model by building in entry and exit and introducing persistence in connections (each of these features introduce challenges for an analytical description of firm volatility). The model starts from an initial firm size distribution, but both the firm size and firm volatility distributions evolve endogenously thereafter.

We show that our simple model with purely homoscedastic shocks goes a long way towards matching the cross-sectional dispersion in firm size and volatility as well as matching the extent of time variation in the aggregate moments of size and volatility. The limitations of the model are also informative: the benchmark version requires more concentration in customer networks than we see in the data in order to fully match the volatility cross section and the sensitivity of firm volatility to size. Therefore, we also study a second model that allows for internal diversification within the boundaries of the firm, compared to the external diversification that we model through the network.

4.1 Calibration

4.1.1 Benchmark Model with Only External Diversification

We start by choosing the parameters listed in Table 4 for our benchmark model, which we refer to as M1.

Size and Volatility Parameters The fundamental volatility of the innovations σ_ε is set to 0.22 and firm growth rate μ_g equals zero. The former allows us to match mean firm volatility, while the latter corresponds to the full-sample growth rate in real market capitalization. The model is initialized with an $N \times 1$ firm size distribution $\boldsymbol{S}_0$, where each $S_{i,0}$ is drawn from

a lognormal distribution with mean $\mu_{s_0} = 10.20$ and standard deviation $\sigma_{s_0} = 1.06$. These numbers are chosen to match the annual cross-sectional moments of the size distribution averaged over all years in our sample.

In each period, a fraction δ of firms die randomly. Each dead firm is replaced by a new firm drawn from the initial firm size distribution.[20] These rules for the connection dynamics and firms' birth and death completely specify the network evolution and the growth rate process in (2). The exogenous firm destruction rate δ is set to 5%, close to the time-series average firm exit rate in our sample of 4.2%.

Network Parameters The parameter γ govern the rate at which idiosyncratic shocks decay as they propagate through the network. The closer γ is to one, the more important higher-order network effects are in determining firms' growth rates. We set $\gamma = 0.95$ to match dispersion of firm volatility as best as possible.[21]

The linkage structure at time 0 is determined by the initial firm size distribution. Each element of the connections matrix $\boldsymbol{B}_0$ is drawn from a Bernoulli distribution with $P(b_{i,j,0} = 1) = p_{i,0}$ as in (4). The probability of forming a supplier-customer connection features the parameter Z, which governs the baseline likelihood of a connection for a firm i, as well as the parameter ζ, which governs how fast that probability decays with the number of firms N. We set $Z = 1.60$ and $\zeta = 0.8$ which implies an expected out-degree of three for a firm of average size. The parameter ζ is chosen close to one so that the expected out-degree grows with N, but at a slower rate.

To induce persistence in links, we modify linkage probabilities to give suppliers a relatively high probability of reconnecting to its customers from the previous period. If firm i did not sell to customer j in period $t-1$, then the probability it does so at time t is simply given by equation (4). If j was a customer of i at $t-1$, then the probability that j continues to be i's customer is $\min\{p_{i,j,t} + \kappa, 1\}$. Thus, κ governs the persistence of connections. We set κ

[20] We have also analyzed versions of the model where there is additional endogenous exit and where new firms are drawn from the lower half of the initial size distribution. Both features have little qualitative effect on our results.

[21] A spatial autoregression of supplier growth rates on the weighted average growth rate of its Compustat customers results in a point estimate for γ of at least 0.90 across a range of specifications. This provides direct evidence on the plausibility of this parameter value.

Table 4: Model Parameters

		M1	M2
	Panel A: Size and Volatility Parameters		
σ_ε	*Fundamental shock volatility*	0.22	0.40
μ_g	*Exogenous firm growth rate*	0.00	0.00
μ_s	*Mean of initial log size distribution*	10.2	10.2
σ_s	*Standard deviation of initial log size distribution*	1.06	1.06
δ	*Exogenous firm death rate*	0.05	0.05
λ	*Governs the internal diversification effect*	0.00	0.90
	Panel B: Network Parameters		
γ	*Governs the network propogation effect*	0.95	0.95
Z	*Governs the probability of new connections*	1.60	0.46
ζ	*Governs the linkage sparsity effect*	0.80	0.80
κ	*Governs persistence in connections*	0.50	0.50
ψ	*Governs the sensitivity of connection weight to customer size*	1.00	0.10

equal to 0.5 to match the observed 54% time-series average death rate of truncated links.[22]

4.1.2 Extended Model with Internal and External Diversification

We consider an extended model, M2, that allows for internal diversification effects by having volatility of a firm's growth rate innovation depend negatively on firm size:

$$\sigma^2_{\varepsilon,t} = \sigma^2_0 - \lambda \frac{\log S_t - E[\log(S)]}{E[\log(S)]}, \tag{9}$$

where σ^2_0 plays the role of σ^2_ε in the benchmark model and the new coefficient λ governs the semi-elasticity of fundamental firm variance with respect to firm size. One motivation for why large firms witness less volatile shocks is internal diversification: when two stand-alone firms merge, the new firm is larger and less volatile because the businesses are not perfectly correlated with one another. We choose $\sigma^2_0 = 0.40$ and $\lambda = 0.90$ to match the cross-sectional mean and standard deviation of the firm volatility distribution, holding fixed all other parameters.

[22] In our benchmark model the average death rate of truncated links is 57% whereas the average death rate of untruncated links is 46%.

The extended model also changes the form of the weighting function $w_{i,j,t}$, which governs the importance of customer j in supplier i's network, by introducing the curvature parameter ψ:

$$w_{i,j,t} = \frac{b_{i,j,t} S_{j,t}^{\psi}}{\sum_{k=1}^{N} b_{i,k,t} S_{k,t}^{\psi}}. \tag{10}$$

By setting $\psi = 0.1$, we make the importance of a given customer less steep in customer size. We lower Z from its benchmark value of 1.60 to 0.46, increasing the expected number of connections for a firm of average size from 3 to 10.

4.2 Simulation Procedure

We simulate our model for 2,000 firms and 1,300 periods (years). We discard the first 300 observations to let the network settle down to its long run distribution, and compute model statistics by averaging over the last 1,000 years. In each period, we report moments based on the sample of the largest 1,000 firms. We focus on the largest firms to account for selection: our Compustat sample is the subset of public firms, which are typically the largest firms in the economy, and these firms are connected to a large number of unobserved smaller firms. Our choice of a total number of $N = 2,000$ firms is dictated by computational considerations.[23]

The number of public firms in our data sample fluctuates over time, but our calibration considers a constant sample of 1,000 public firms. To further improve comparability between the model and data, we focus on the top 33% of Compustat firms by size, which in our data contains 1,000 firms on average. For completeness, we also report the empirical moments for the entire distribution of public firms (about 3,000 firms on average).

Our simulated model is well suited to deal with truncation. We implement the truncation inside the model to match the truncation of the data. That is, we treat $w_{i,j,t}$ as unobserved whenever $w_{i,j,t}$ is below 10% in the simulation. Since we can observe both truncated and untruncated moments of the model as calibrated to the data, we are able to make indirect inferences about the behavior of the full, untruncated economy.

[23] The same random number generator seeds are used for each model so that, when comparing models, differences in results are due only to differences in parameters rather than differences in random shock draws.

Finally, in order to compare variance moments in the model and data, we take into account the fact that empirical variances are estimated with noise.[24]

4.3 Calibration Targets and Simulation Results

Below, we outline the features of the observed size, volatility, and network data that we target in our calibration and discuss how well the benchmark model fits them.

4.3.1 Size Distribution

Table 5 reports the target moments of the cross-sectional log size distribution. All reported moments are time-series averages over the 1926 to 2010 sample for size and volatility data, and over 1980 to 2010 for sales network data. The first column reports moments for the full cross section of firms observed in Compustat. The second column reports results for the top 33% of Compustat firms by size in each year. Column (2) is the main column of interest in the data. For example, 11.63 is the average log firm size of the large-firm cross section (this is naturally higher than the 9.61 mean for the full Compustat sample). The dispersion of the log size distribution is 1.06 for the top-33% group and 1.79 for the full sample. As can be seen from column (3), the benchmark model (M1) matches the main moments of the firm size distribution. It matches the cross section mean and dispersion in size exactly, by virtue of the calibration. The other moments of the size distribution constitute over-identifying restrictions. The inter-quartile range for log firm size is [10.79,12.25], versus [10.78,12.25] in the top-33% sample. At both extremes of the size distribution, the model also compares favorably to the data. Since the size distribution evolves endogenously, the network effects are crucial to generate enough heterogeneity in firm size. Firms with well-diversified customer networks are better diversified and have lower volatility, which limits the variance of their growth rate and hence their size next period. On the other hand, firms with concentrated customer networks can increase or decrease in size by a lot between periods.

Panel B reports time series properties of the size distribution. The first two rows show

[24] We report "estimated" variance moments in the model, which we compute as $\log(\widehat{Var}_t[g_{t+1}]) = \log(Var_t[g_{t+1}]) + e_{t+1}$, where $e \sim \mathcal{N}(0, \sigma_e^2)$ and σ_e is the time series average of the cross-sectional standard deviation of $\log(Var_t[g_{t+1}])$. Our results are qualitatively insensitive to the choice of σ_e.

that size dispersion has high variability over time and is highly persistent. Its time-series standard deviation is 14% for the top-33% firms and 24% for all firms. The third row shows that the cross-sectional standard deviation of size *growth* (log size changes) is also volatile over time. The time series standard deviation is 15% for all firms and 11% for the top-33% firms. These results confirm that the size distribution moves around considerably over time. In the benchmark model, size dispersion also moves around substantially over time and is highly persistent. The model produces the same high time-series variability in the size dispersion as observed in the data (19% compared to 14% in column (2) and 24% in column (1)), but understates the dispersion in growth rates (5% versus 11% in column (2) and 15% in column (1)). The network structure is crucial for generating such a high degree of time series variation in the size distribution. Shutting down the network effects in this calibration (setting $\gamma = 0$) reduces aggregate size distribution dynamics to essentially zero.

4.3.2 Volatility Distribution

Table 6 reports moments for distribution of log variance. Columns (1) and (2) report market-based log variance while columns (3) and (4) report sales-based log variance. Because volatilities (standard deviations in levels) are more intuitive than log variances, we exponentiate the moments of log variance and then take their square root. All reported moments are time-series averages unless explicitly mentioned otherwise. Panel A shows that average market-based volatility is 30% per year for the large-firm sample and 41% per year for the full cross-section. Average sales-based volatility is 24% for the large-firm sample and 30% for the full sample. The range of return-based (sales-based) volatilities goes from 17% (8%) at the 5^{th} percentile to 54% (77%) at the 95^{th} percentile for the latter group. The cross-sectional dispersion in return volatility is 73% in column (2) and 97% in the full cross section reported in column (1). Sales-based volatility has even larger dispersion of 152% and 142% in columns (3) and (4), respectively (in part because sales-based volatility is measured with more noise).

Column (5) of Table 6 shows that model M1 generates high average firm volatility of 37%, in between the return volatility in the full sample and that in the top-33% sample, and is capable of generating a wide range of firm volatility. The cross-sectional dispersion

Table 5: Firm Size Distribution Target Moments

	All	Top-33%	M1	M2
	Panel A: Cross-sectional Moments of Log Size			
Avg	9.61	11.63	11.63	12.03
SD	1.79	1.06	1.07	1.03
5%	6.86	10.38	10.32	10.69
10%	7.39	10.47	10.44	10.82
25%	8.33	10.78	10.79	11.19
Med	9.48	11.39	11.42	11.86
75%	10.77	12.25	12.25	12.71
90%	12.05	13.10	13.10	13.48
95%	12.76	13.64	13.68	13.92
	Panel B: Time Series Properties of Size Distribution			
SD of $\sigma_{S,t}$	0.24	0.14	0.19	0.44
AR(1) $\sigma_{S,t}$	0.947	0.939	0.995	0.998
SD of $\sigma_{g,t}$	0.15	0.11	0.05	0.16

Notes: All reported moments in Panels A and B are time-series averages of the listed year-by-year cross sectional moments (cross section average, standard deviation, and percentiles) for the sample 1926–2010. The first column reports the full cross section of firms . The second column reports results for the top-33% of firms in each year. Column (3) reports the corresponding moments for the benchmark model M1. Column (4) reports the moments for the extended model M2. Panel A reports moments of the log size distribution, where size is defined in the data as market equity. Panel B reports the time-series standard deviation and time-series persistence of size dispersion $\sigma_{S,t}$, defined as the cross-sectional standard deviation of log size, and the time-series standard deviation of the cross section standard deviation of log growth rates $\sigma_{g,t}$.

in volatility is 45%, compared to 73% for the return-based measure in column (2). The model matches the 90^{th} and 95^{th} percentiles of volatility, but overestimates volatility on the low end of the distribution. The least volatile firms have a volatility of 26% in the model, but only 17% in the return data. Network effects are again crucial in generating dispersion in volatility outcomes. For $\gamma = 0$, all firms' volatilities would be identical. So, while the benchmark model cannot generate all of the observed dispersion in firm volatility, it can generate a substantial share of it and that is entirely due to the network effects.

In Panel B we compute the cross-sectional correlation between log size at time t and log variance at time $t+1$ for each year t, then report the average across all years. Similarly, the second row reports the slope coefficient (beta) of a cross-sectional regression of log variance

Table 6: Firm Volatility Distribution Target Moments

	All Returns	Top-33% Returns	All Sales	Top-33% Sales	M1	M2
	Panel A: Cross-Sectional Moments of Firm Volatility					
Avg	0.41	0.30	0.30	0.24	0.37	0.36
SD	0.97	0.73	1.52	1.42	0.45	0.71
5%	0.19	0.17	0.09	0.08	0.26	0.20
10%	0.22	0.19	0.11	0.10	0.28	0.23
25%	0.29	0.24	0.17	0.14	0.31	0.28
Med	0.40	0.30	0.29	0.23	0.36	0.36
75%	0.56	0.38	0.49	0.38	0.42	0.46
90%	0.76	0.48	0.78	0.60	0.49	0.57
95%	0.91	0.54	1.07	0.77	0.54	0.65
	Panel B: Cross-Sectional Co-moments of Size and Volatility					
$Corr(S_t, V_{t+1})$	-0.58	-0.33	-0.34	-0.18	-0.42	-0.62
$\beta(S_t, V_{t+1})$	-0.32	-0.23	-0.28	-0.22	-0.18	-0.46
	Panel C: Time Series Co-moments of Size and Volatility					
SD of $\mu_{\sigma,t}$	0.69	0.64	0.46	0.42	0.20	0.52
SD of $\sigma_{\sigma,t}$	0.18	0.12	0.13	0.13	0.05	0.11
$Corr(\sigma_{S,t}, \mu_{\sigma,t})$	0.72	0.55	0.53	0.51	0.95	0.90
$Corr(\sigma_{S,t}, \sigma_{\sigma,t})$	0.79	0.76	0.60	0.38	0.77	0.91
SD of $g_{agg,t}$	0.21	0.20	0.21	0.20	0.20	0.23

Notes: All reported moments in Panels A and B are time-series averages of the listed year-by-year cross sectional moments (cross section average, standard deviation, and percentiles) for the sample 1926–2010. The first and third columns report return volatility and sales growth volatility, respectively, for the full cross section of firms. The second and fourth columns report results for the largest 33% of firms by market capitalization in each year. Annual variances in Columns (1) and (2) are calculated from daily stock return data, while variances in columns (3) and (4) are based on 20 quarters of sales growth rates (in the current and the next four years). Moments are computed based on the log variance distribution, but are expressed as volatilities in levels for exposition. That is, we exponentiate each moment of the log distribution and take the square root. Column (5) reports the corresponding moments for the benchmark model M1. Column (6) reports the moments for the extended model M2. Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.

at time $t+1$ on a constant and log size at time t. Both are strongly negative in the data showing that large firms have lower volatility over the next period. The benchmark model generates correlation of -42% between size and volatility at the firm level, splitting the correlations of -33% and -58% in the two data samples. The slope of the relationship between variance and size is slightly smaller than that in the data (-18% versus -23% for

the largest firms). Given i.i.d. shocks, the negative correlation arises endogenously from the network effect: smaller firms have fewer connections, hence a more concentrated network of customers, and higher volatility.

Panel C reports time series properties of aggregate moments of the volatility distribution and the joint size-volatility distribution. The first row shows that the time-series standard deviation of average firm variance is 64% in the top-33% sample and 69% in the full sample. Average variance is highly variable over time. The second row shows that the cross-sectional dispersion of variance also moves substantially over time in both samples and for both ways of measuring volatility. The time series standard deviation is 12% in the top-33% sample. The model also generates substantial variability over time in the cross-sectional mean of firm variance (20%) and in its dispersion (5%). The third and fourth rows report two key moments in our paper (see Figures 1 and 2). First, the time-series correlation between size dispersion at t and mean volatility at time $t+1$ are strongly positively correlated in the data (72% in the full sample and 55% in the large-firm sample) as well as in the model (95%). Second, size dispersion is strongly positively correlated with volatility dispersion in the data (79% in the full sample and 76% in the large-firm sample) as well as in the model (77%). The last row reports the volatility of aggregate growth, computed as the volatility of the size-weighted average of the relevant firm sample, which is 21% for all public firms and 20% for the top-33% sample. It also is 20% in the model. The model produces substantial time series volatility of the cross section moments, though it is somewhat understated in some cases. All fluctuations in the volatility distribution rely crucially on the network structure of the model. As with the size distribution, shutting down network effects in the simulations produces a volatility distribution whose moments are nearly constant over time. Thus, our model with only i.i.d. homoscedastic shocks endogenously generates periods with more uncertainty and periods with less uncertainty. "High uncertainty" periods are associated with a high level of concentration in the firm size distribution.

Finally, we re-estimate the volatility factor regressions of Table 2 on simulated data from the model. The corresponding R^2 statistics for panel regressions of total volatility are 36%, 37%, and 40% for the three factors considered in that table (from left to right). These numbers are quite close to their empirical counterparts of 24%, 26%, and 39%. Thus, the

Table 7: Network Target Moments

	All Returns	Top-33% Returns	All Sales	Top-33% Sales	M1	M2
			Panel A: Out-degree Moments			
median N^{out}	1.00	1.00	–	–	1.71	0.48
99^{th} % N^{out}	3.38	3.32	–	–	4.66	7.39
median H^{out}	0.05	0.03	–	–	0.60	0.12
99^{th} % H^{out}	0.95	0.54	–	–	1.00	0.56
$Corr(N_t^{out}, S_t)$	0.01	−0.07	–	–	0.34	−0.45
$Corr(H_t^{out}, S_t)$	−0.31	−0.08	–	–	−0.71	−0.54
$Corr(H_t^{out}, V_{t+1})$	0.15	0.08	0.29	0.10	0.54	0.43
			Panel B: In-degree Moments			
median N^{in}	1.00	1.12	–	–	1.90	1.90
99^{th} % N^{in}	31.61	16.80	–	–	7.39	6.28
median H^{in}	0.00	0.00	–	–	0.11	0.01
99^{th} % H^{in}	0.28	0.24	–	–	0.85	0.05
$Corr(N_t^{in}, S_t)$	0.37	0.49	–	–	0.49	0.15
$Corr(H_t^{in}, S_t)$	−0.26	−0.20	–	–	0.03	0.02
$Corr(H_t^{in}, V_{t+1})$	0.13	0.08	0.08	0.08	0.01	−0.00

Notes: All reported moments are time-series averages unless explicitly mentioned otherwise. The first and second columns report data for the cross section of firms for which we have customer information from Compustat. The sample is 1980-2009. The variance in Column (1)-(2) are calculated based on daily stock return data, while the variance in Column (3)-(4) are based on 20 quarters worth of annual sales growth (in the current and the next four years). Column (5) reports the corresponding moments for the benchmark model M1. Column (6) reports the moments for the extended model M2. Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data. All model calculations impose the 10% truncation on network weights $w_{i,j}$ uniformly.

model quantitatively replicates the factor structure in volatilities.[25]

4.3.3 Network Moments

Table 7 reports calibration results for network connectivity and concentration. Panel A focuses on out-degrees and the bottom panel focuses on in-degrees.

[25]In untabulated analyses we run Granger Causality tests of volatility moments on lagged size dispersion in the simulated model and find very similar results to Granger tests on actual data reported in Appendix B.3.

Out-Degrees Column (5) of Table 7 shows the network-related properties of the benchmark model. Like the data in columns (1) and (2), the model features a small number of supplier-customer relationships after truncation: the median is 1.71 in the model and 1.00 in the data. The 99^{th} percentile of the truncated out-degree distribution is 4.66 in the model compared to 3.32 in the data. Truncation severely affects the out-degrees: the median of the *untruncated* out-degree distribution is 3.71 in the model while the 99^{th} percentile is 112.[26]

Next we turn to customer network concentration or out-Herfindahls. The Herfindahl indices, which are also based on truncated degree information, are less biased because large customers receive a large weight and are more likely to be in the database. The main shortcoming of the benchmark calibration is that customer network concentration (out-Herfindahl) is too high. The median is 0.60, higher than the 0.05 value we observe in the data. The reasons are that in M1 suppliers typically have a small number of connections and the weighting function is linear in customer size, making large customers extremely important in a suppliers network. To generate substantial dispersion in firm volatilities in a setting with only uncorrelated shocks, the model requires too much concentration in customer networks. The extended model M2 discussed below addresses this shortcoming.

The fifth row of Panel A reports the correlation between out-degree and supplier size. As discussed in Section 2.6, we find a nearly zero correlation between truncated out-degree and size in the data. Because of truncation in the network data, the zero correlation based on truncated degrees belies a substantial positive correlation between size and true, untruncated out-degrees. Our simulation results show that large firms have many more connections than small firms, with a cross-sectional correlation of 34% between the truncated out-degree and log size in the model, and a 60% correlation between *untruncated* out-degree and log size. This implies that truncation downward biases the correlation estimate by 26 percentage points. Using the model-implied bias to correct the observed correlations in the data (1% for the full sample and -7% for the top-33% sample) implies a 19% to 27% correlation between untruncated out-degree and log size in the data.

Two moments that lie at the heart of the model's mechanism are the cross-sectional

[26] To the extent the data permit, Appendix B.1 discusses the effect of truncation on the distributions of the out-degrees and the out-Herfindahls in the data.

correlations of the out-Herfindahl with size and of the out-Herfindahl with variance. The model generates a strong negative correlation between out-degree Herfindahl and size (-71%) and a positive correlation between out-degree Herfindahl and volatility (54%), arising because large firms have a better diversified and thus less concentrated customer bases. In the data, we find a -31% correlation between size and out-Herfindahl. We also find a positive relationship between supplier log variance and out-Herfindahl: the correlation is 15% based on returns and 29% based on sales. That is, firms with a more diversified customer network indeed experience lower volatility because they are better insulated against a shock to any single customer. Appendix B.1 shows that these three correlations are statistically significant at the 1% level. Similar to the firm-level evidence, it also finds a -39% correlation between an industry's log size and its out-Herfindahl and a 22% correlation between its out-Herfindahl and log volatility.

In-Degrees Turning to Panel B of Table 7, the median truncated number of suppliers, or in-degree, is 1 while the 99^{th} percentile is 17 or 32 depending on the data sample. In the benchmark model, the median in-degree is 1.90 and the 99^{th} percentile is between 6 and 8. Concentration of supplier networks is lower than that of customer networks, but still too high relative to the data.

In the model, the probability that a connection exists between supplier i and customer j depends only on i's size and is independent of firm j's size. Thus, the model predicts a zero correlation between size and in-degree in the *absence* of link truncation. While we assume smaller firms are equally likely to be customers as large firms, truncation will eliminate the links with small customers, leaving large firms counted as customers far more often. Thus we expect to see a strong association between size and in-degree in the truncated data (we find a correlation of 37%). Indeed, the benchmark model M1 produces a 49% correlation between size and in-degree when the simulated samples are truncated. However, the correlation between *untruncated* in-degree and size is 4%, which implies that the 37% correlation estimate in the data is severely upward biased by truncation. The magnitude of the bias is such that the bias-corrected correlation between *untruncated* in-degree and log size in the data is close to zero for both data samples, and provides evidence that the

likelihood of a connection between a supplier and a customer does not depend strongly on the customer's size, as assumed in our model. Finally, we find a modestly positive correlation between in-Herfindahl and both measures of volatility, though these estimates are statistically insignificant in the data (t-statistics shown in the appendix).

4.3.4 Extended Model with Internal and External Diversification

The benchmark model calibration with only external (network) diversification implies too much concentration and fails to generate enough dispersion in firm volatility. The extended model (M2) with internal diversification jointly matches size, volatility, *and* network moments. This model can generate a wider firm volatility spread than the benchmark model while continuing to generate a firm size distribution that is similar to the data. In particular, M2 generates time-series correlations between firm size dispersion and the mean and dispersion of firm variance that are qualitatively similar to the data. Average firm volatility has a 52% standard deviation, compared to 20% in M1 and 64% in the data. In general, M2 produces much more action in aggregate moments of the size and volatility distributions. Firm size and volatility are cross-sectionally more negatively correlated with each other in M2 than in M1, bringing the model closer to the data, noting that part of this correlation is now mechanically built in.

Most importantly, the truncated network moments from M2 are much closer to the data than those of M1. The median out-Herfindahl is 0.12, a factor of 5 smaller than in M1, and much closer to the data. In addition, the truncated number of suppliers (in-degree) and the supplier network concentration (in-Herfindahl) are a closer fit with the data than the benchmark model.[27] Thus, the model matches the key features of the size and volatility distributions while respecting the observed degree of concentration in customer and supplier networks.

The internal diversification effect, modeled as an exogenous negative correlation between shock volatility and firm size, cannot match the data in the absence of network effects. In

[27]While the correlation between log size and truncated out-degree (number of customers) is -45% in M2 in row 5 of Panel A, the correlation between log size and untruncated out-degree is $+74\%$. Thus, the truncation bias for this correlation is much larger in M2 than in M1. The bias in the correlation between log size and in-degree (number of suppliers) is smaller in M2 than in M1; the correlation between log size and untruncated in-degrees is similar in both models (0.06 in M2 and 0.04 in M1).

particular, the correlation between firm size dispersion and mean firm volatility turns from positive to negative once we shut down the network effects in M2 (setting $\gamma = 0$). Without network transmission, an increase in the firm size dispersion makes the firms in the top half of the distribution (public firms) larger, which lowers their average volatility by assumption. Hence, the network effects are *solely* responsible for the strong positive correlation between size dispersion and mean volatility, our main moment of interest. This result underscores the crucial role of network effects in accounting for the firm volatility facts, over and above internal diversification effects.

4.3.5 Downstream Transmission

Our paper also raises an interesting question on the direction of shock propagation in economic networks. The literature studies how inter-sector trade networks influence aggregate volatility and typically considers *downstream* transmission of shocks from intermediate goods producers to final goods producers. Our results suggest that upstream shock propagation provides a better description of firm volatility data, as in equilibrium models with final demand shocks (Shea (2002)).

To better understand the difference in predictions, Appendix C studies a version of our model with downstream transmission of shocks instead of upstream transmission. All parameters are the same as those in M1. Interestingly, the downstream model delivers identical size and volatility moments, highlighting the usefulness of network mechanisms for describing joint size and volatility facts, regardless of the direction of shock propagation.

The network moments, however, help identify the transmission direction. The in-degree and out-degree moments are reversed between upstream and downstream models.[28] The downstream model matches the correlations between in-Herfindahl and size and volatility better, but completely misses the correlation between out-Herfindahl on the one hand and size and volatility on the other hand. In the data, the correlation between volatility and out-Herfindahl is statistically much stronger than that with in-Herfindahl, as in the model with upstream shock transmission.

[28] The statement is true for untruncated in-degree and out-degree moments. Truncation affects the network moments; see appendix for a detailed discussion.

5 Conclusion

We document new features of the joint evolution of the firm size and firm volatility distribution and propose a new model to account for these features. In the model, shocks are transmitted upstream from customers to suppliers. Firms sell products to an imperfectly diversified portfolio of customers. The larger the supplier, the more customer connections the supplier has, the better diversified it is and the lower its volatility. Large customers have a relatively strong influence on their suppliers, so shocks to large firms have an important effect throughout the economy.

When the size dispersion increases in this economy, large firms become more important and many customer networks become less diversified. In those times, average firm volatility is higher as is the cross-sectional dispersion of volatility. Because the underlying innovations are i.i.d. over time, the model endogenously generates "uncertainty shocks". We provide direct evidence of network linkages and use supplier-customer relationship data to discipline the calibration of our model. A calibrated version of the network model augmented with internal diversification effects quantitatively replicates the most salient features of the firm size and the volatility distributions; without network effects, the model cannot match the data.

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A Theoretical Appendix

A.1 Proofs of Propositions

Proof of Lemma 1:

Proof. Note that sample moments of $\{B_{i,k}S_k\}_{k=1}^N$ are equivalent to the sample moments of a random sample of N_i elements from the size distribution, where $N_i = \sum_k B_{i,k}$. If S_i has finite variance, then the LLN implies that

$$N_i H_i = \frac{N_i^{-1}\sum_k B_{i,k}S_k^2}{\left(N_i^{-1}\sum_k B_{i,k}S_k\right)^2} \xrightarrow{a.s.} \frac{E[S^2]}{E[S]^2}.$$

Because N_i is a sum of *iid* bernoulli variables, $N_i \sim Np_i$. this implies that $N_i/N^{(1-\zeta)} \xrightarrow{a.s.} \tilde{S}_i/Z$, which delivers the result. Under log-normality, $E[S^2]/E[S]^2 = e^{\sigma_s^2}$. □

Proof of Lemma 2:

Proof. We begin with the case $q = 2$.

$$[W^2]_{i,j} = \sum_k w_{i,k}w_{k,j} = \sum_k \left(\frac{B_{i,k}S_k}{\sum_m B_{i,m}S_m}\frac{B_{k,j}S_j}{\sum_l B_{k,l}S_l}\right) = \left(\frac{S_j}{\sum_m B_{i,m}S_m}\right)\left(\sum_k \frac{B_{i,k}B_{k,j}S_k}{\sum_l B_{k,l}S_l}\right).$$

As in the previous lemma, the LLN implies $N_i^{-1}\sum_k B_{i,k}S_k \xrightarrow{a.s.} E[S]$. This implies

$$\frac{S_j}{\sum_m B_{i,m}S_m} \sim \frac{\tilde{S}_j}{\tilde{S}_i}\frac{Z}{N^{1-\zeta}}$$

To characterize the asymptotic behavior of the second term, first note that Markov's LLN for heterogeneously distributed variables implies that $N^{-(1-2\zeta)}\sum_k B_{i,k}B_{k,j} \xrightarrow{a.s.} \tilde{S}_i/Z^2$.[29] This in turn implies that

$$\sum_k \frac{B_{i,k}B_{k,j}S_k}{\sum_l B_{k,l}S_l} \sim \sum_k \frac{B_{i,k}B_{k,j}S_k}{\frac{S_k}{Z}N^{1-\zeta}} \sim \frac{Z}{N^{1-\zeta}}Np_iE[p_k] = N^{-\zeta}\frac{\tilde{S}_i}{Z}.$$

Together with the asymptotic behavior of the first term we have

$$[W^2]_{i,j} \sim \frac{\tilde{S}_j}{N}.$$

For induction, assume true for $q > 2$, so that

$$[WW^q]_{i,j} \sim \sum_k \frac{B_{i,k}S_k\tilde{S}_j}{N\left(\sum_l B_{i,l}S_l\right)} \sim \frac{\tilde{S}_j}{N}\frac{\sum_k B_{i,k}S_k}{\sum_l B_{i,l}S_l} \sim \frac{\tilde{S}_j}{N}.$$

This quantity is of the same form as $[W^2]_{i,j}$, and the same rationale applied in that case gives $[W^{q+1}]_{i,j} \sim \tilde{S}_j/N$.

We also have that

$$[W\bar{W}']_{i,j} = \sum_k \frac{B_{i,k}S_k^2}{\left(\sum_l B_{i,l}S_l\right)NE[S]} \sim \frac{E[S^2]}{NE[S]^2}$$

because the finite variance of S_k and the LLN imply $N_i^{-1}\sum_k B_{i,k}S_k \xrightarrow{a.s.} E[S]$ and $N_i^{-1}\sum_k B_{i,k}S_k^2 \xrightarrow{a.s.} E[S^2]$.

[29] See Theorem 3.7 of White (2001). For Markov's LLN to apply, the so-called Markov condition must hold. Applied to the current setting, this requires $\sum_{k=1}^{\infty} E|B_{k,j} - p_k|^{1+\delta}/k^{1+\delta} < \infty$. Because $B_{k,j}$ are independent Bernoulli draws this condition is satisfied.

Similarly,

$$[\bar{W}\bar{W}']_{i,j} = \sum_k \frac{\tilde{S}_k^2}{N^2} \sim \frac{E[S^2]}{NE[S]^2}.$$

Note that $H_i = [WW']_{ii}$ and Lemma 1 applies. For off-diagonal elements of WW',

$$[WW']_{i,j} = \frac{\sum_k B_{i,k}B_{j,k}S_k^2}{\left(\sum_l B_{i,l}S_l\right)\left(\sum_l B_{j,l}S_l\right)} \sim \frac{E[S^2]}{NE[S]^2}.$$

□

Proof of Proposition 1

Proof. Because $V(g) = \sigma_\varepsilon^2(I-\gamma W)^{-1}(I-\gamma W')^{-1}$, we study the behavior of $(I-\gamma W)^{-1}$ as the number of firms N becomes large. Noting that $(I-\gamma W)^{-1} = I+\gamma W+\gamma^2 W^2+...$, Lemma 2 establishes the asymptotic equivalence

$$(I-\gamma W)^{-1} \sim I + \gamma W + \frac{\gamma^2}{1-\gamma}\bar{W}.$$

The outer product of $I+\gamma W+\frac{\gamma^2}{1-\gamma}\bar{W}$ is

$$I+\gamma W+\gamma W'+\gamma^2 WW'+\frac{\gamma^2}{1-\gamma}\bar{W}+\frac{\gamma^2}{1-\gamma}\bar{W}'+\frac{\gamma^3}{1-\gamma}W\bar{W}'+\frac{\gamma^3}{1-\gamma}\bar{W}W'+\frac{\gamma^4}{(1-\gamma)^2}\bar{W}\bar{W}' \tag{11}$$

From Lemmas 1 and 2, the behavior of the i^{th} diagonal element of $V(g)$ in a large economy is described by the stated asymptotic equivalence. In the lognormal special case, $E[S^2]/E[S]^2 = \exp(\sigma_s^2)$. □

Proof of Proposition 2

Proof. As in the finite variance case, H_i determines the rate of convergence for firm variance. Recall the expression for a firm's Herfindahl:

$$H_i = \sum_k \frac{B_{i,k}S_k^2}{\left(\sum_l B_{i,l}S_l\right)^2} = \frac{N_i^{2/\eta}N_i^{-2/\eta}\sum_k B_{i,k}S_k^2}{N_i^2\left(N_i^{-1}\sum_l B_{i,l}S_l\right)^2}$$

From Gabaix (2011, Proposition 2) we have that

$$N_i^{-2/\eta}\sum_k B_{i,k}S_k^2 \xrightarrow{d} u$$

where u is a Levy-distributed random variable. Because $\eta > 1$, mean size is finite and $N_i^{-1}\sum_l B_{i,l}S_l \xrightarrow{a.s.} E[S]$. Therefore,

$$N_i^{2(1-1/\eta)}H_i = \frac{N_i^{-2/\eta}\sum_k B_{i,k}S_k^2}{N_i^{-1}\sum_k B_{i,k}S_k} \xrightarrow{d} \frac{u}{E[S]}.$$

Finally, recall that

$$N_i \sim N^{1-\zeta}\frac{\tilde{S}_i}{Z}$$

Combining gives us the result:

$$N^{(1-\zeta)(2-2/\eta)}H_i \xrightarrow{d} \frac{u}{E[S]}\left(\frac{Z}{\tilde{S}_i}\right)^{2-2/\eta}.$$

□

Proof of Proposition 3

Proof. Because H_i determines the rate of convergence for firm variance, we may understand how firm variances covary in a large economy by studying the asymptotic covariance among H_i and H_j.

Define $\hat{E}_i[S_k^2] = N_i^{-1}\sum_k B_{i,k}S_k^2$. We first characterize the asymptotic behavior of

$$Cov\left(\hat{E}_i[S_k^2], \hat{E}_j[S_k^2]\right) = E\left[N_i^{-1}N_j^{-1}\sum_k\sum_l B_{i,k}B_{j,l}S_k^2S_l^2\right] - E\left[N_i^{-1}\sum_k B_{i,k}S_k^2\right]E\left[N_j^{-1}\sum_k B_{j,k}S_k^2\right].$$

Because size has finite fourth moment, the LLN implies that

$$N^{-(1-2\zeta)}\sum_k B_{i,k}B_{j,k}S_k^4 \xrightarrow{a.s.} E[S^4]\tilde{S}_i\tilde{S}_j/Z^2,$$

$$N^{-(1-2\zeta)}(N-1)^{-1}\sum_{k\neq l} B_{i,k}B_{j,l}S_k^2S_l^2 \xrightarrow{a.s.} E[S^2]^2\tilde{S}_i\tilde{S}_j/Z^2,$$

and

$$N^{-(1-\zeta)}\sum_k B_{i,k}S_k^2 \xrightarrow{a.s.} E[S^2]\tilde{S}_i/Z.$$

These imply that

$$E\left[N_i^{-1}N_j^{-1}\sum_k\sum_l B_{i,k}B_{j,l}S_k^2S_l^2\right] \sim N^{1-2\zeta}\frac{E[S^4]\tilde{S}_i\tilde{S}_j}{N^2p_ip_jZ^2} + (N-1)N^{1-2\zeta}\frac{E[S^2]^2\tilde{S}_i\tilde{S}_j}{N^2p_ip_jZ^2} = \frac{1}{N}E[S^4] + \frac{N-1}{N}E[S^2]^2$$

and

$$E\left[N_i^{-1}\sum_k B_{i,k}S_k^2\right]E\left[N_j^{-1}\sum_k B_{j,k}S_k^2\right] \sim N^{2(1-\zeta)}\frac{E[S^2]^2\tilde{S}_i\tilde{S}_j}{N^2p_ip_jZ^2} = E[S^2]^2$$

so that

$$Cov\left(\hat{E}_i[S_k^2], \hat{E}_j[S_k^2]\right) \sim N^{-1}V(S^2).$$

Since $H_i = N_i^{-1}\hat{E}_i[S_k^2]/\left(N_i^{-1}\sum_k B_{i,k}S_k^2\right)$, we have

$$Cov(H_i, H_j) \sim \frac{1}{N^{2(1-\zeta)}}\frac{Z^2}{\tilde{S}_i\tilde{S}_jE[S]^4}Cov\left(\hat{E}_i[S_k^2], \hat{E}_j[S_k^2]\right)$$

which delivers the stated asymptotic equivalence. □

Proof of Proposition 4

Proof. Let $\boldsymbol{S}_t$ be the vector of sizes, and $\boldsymbol{\nu}_t = \boldsymbol{S}_t/\iota'\boldsymbol{S}_t$ the vector of size weights.

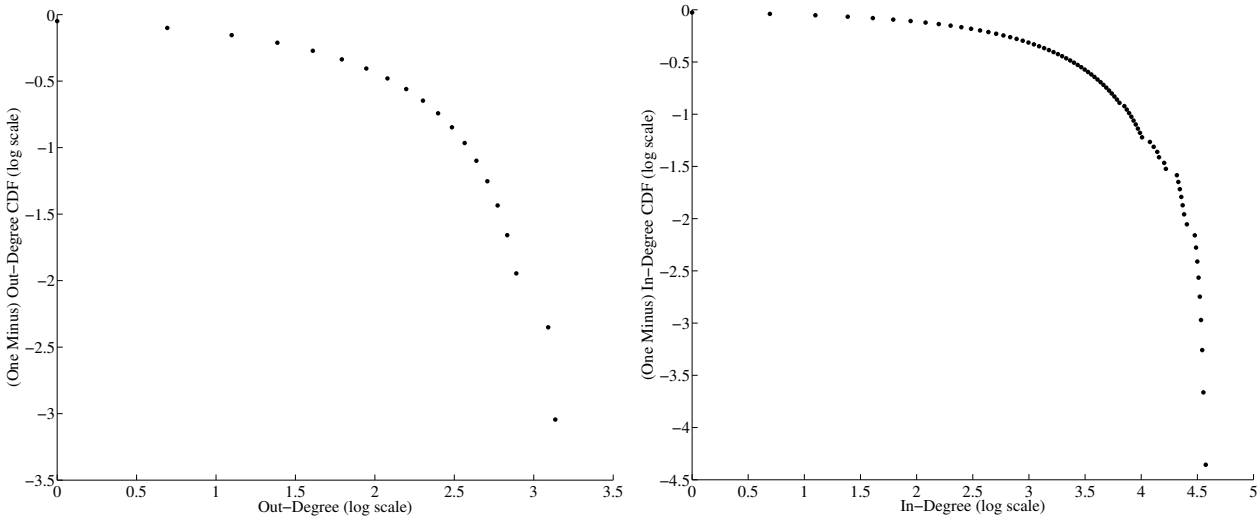
$$g_{a,t+1} = \boldsymbol{\nu}_t'\boldsymbol{g}_{t+1} = \boldsymbol{\nu}_t'\left(\boldsymbol{I} - \gamma\boldsymbol{W}_t\right)^{-1}\left(\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}_{t+1}\right).$$

The variance of the aggregate growth rate is given by:

$$\begin{aligned} V_t(g_{a,t+1}) &= \sigma_\varepsilon^2\boldsymbol{\nu}_t'\left(\boldsymbol{I} - \gamma\boldsymbol{W}_t\right)^{-1}\left(\boldsymbol{I} - \gamma\boldsymbol{W}_t'\right)^{-1}\boldsymbol{\nu}_t, \\ &\sim \sigma_\varepsilon^2\left[\boldsymbol{\nu}_t'\boldsymbol{\nu}_t + \gamma\boldsymbol{\nu}_t'W\boldsymbol{\nu}_t + \gamma\boldsymbol{\nu}_t'W'\boldsymbol{\nu}_t + \gamma^2\boldsymbol{\nu}_t'WW'\boldsymbol{\nu}_t + \frac{\gamma^2}{1-\gamma}\boldsymbol{\nu}_t'\bar{W}\boldsymbol{\nu}_t + \frac{\gamma^2}{1-\gamma}\boldsymbol{\nu}_t'\bar{W}'\boldsymbol{\nu}_t\right. \\ &\quad \left. + \frac{\gamma^3}{1-\gamma}\boldsymbol{\nu}_t'W\bar{W}'\boldsymbol{\nu}_t + \frac{\gamma^3}{1-\gamma}\boldsymbol{\nu}_t'\bar{W}W'\boldsymbol{\nu}_t + \frac{\gamma^4}{(1-\gamma)^2}\boldsymbol{\nu}_t'\bar{W}\bar{W}'\boldsymbol{\nu}_t\right], \end{aligned} \tag{12}$$

where the second expression is asymptotically equivalent, as explained in Proposition 1. Define $\bar{H} = E[S^2]/(NE[S]^2)$. From the previous derivations, the following equivalencies hold: $\boldsymbol{\nu}'\iota = 1, \boldsymbol{\nu}'\boldsymbol{\nu} \sim \bar{H}, \boldsymbol{\nu}'W \sim$

Figure 4: Customer-Supplier Network Degree Distributions



Notes: The figure plots log-log survivor plots of the out-degree and in-degree distributions of the Compustat customer-supplier network data pooling all firm-years for 1980–2009.

$\boldsymbol{\nu}', \boldsymbol{\nu}'W' \sim \bar{H}\iota', \bar{W}'\boldsymbol{\nu} \sim \boldsymbol{\nu}$, and $\bar{W}\boldsymbol{\nu} \sim \iota\bar{H}$. Applying these to (12) and combining terms, we find that the aggregate variance is asymptotically equivalent to:

$$V_t(g_{a,t+1}) \sim \sigma_\varepsilon^2 \left[1 + 2\gamma + \gamma^2 + 2\frac{\gamma^2}{1-\gamma} + \frac{2\gamma^3}{1-\gamma} + \frac{\gamma^4}{(1-\gamma)^2}\right] \bar{H} = \frac{\sigma_\varepsilon^2}{(1-\gamma)^2}\bar{H}$$

which delivers the stated result. □

B Empirical Appendix

This appendix discusses several additional empirical results.

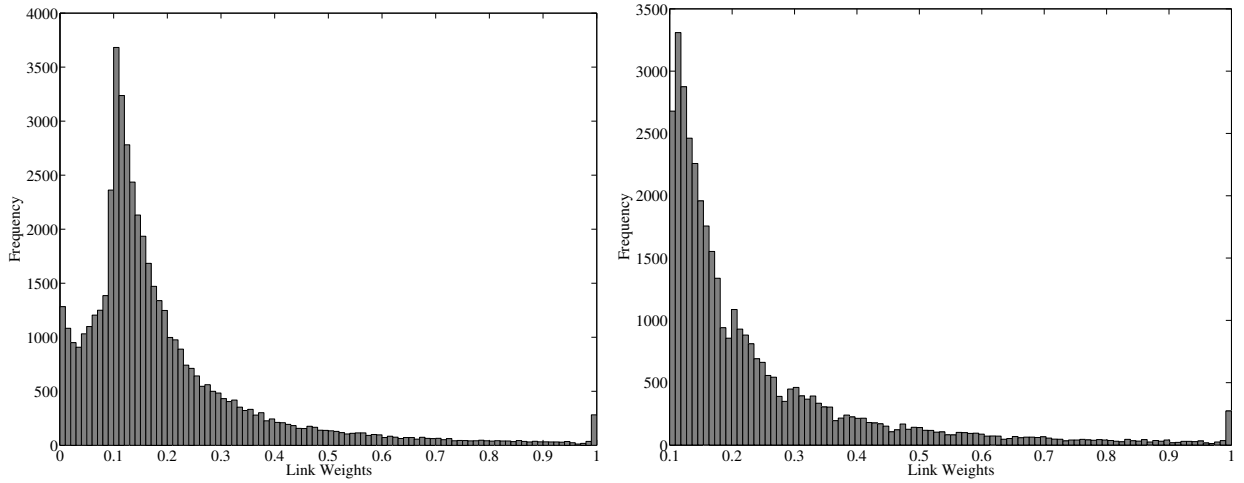
B.1 Summary Statistics Network Data

This appendix provides additional detail on the network data.

We find that the number of customers a Compustat firm has, the "out-degrees," ranges between 1 and 24 while the number of suppliers a firm has, the "in-degrees," ranges between 1 and 130. Firms can but are not required to report customers that represent less than 10% of their sales. Out-degrees can reach 24 since some suppliers (23%) voluntarily report customers that fall below the 10% sales threshold. The maximum out-degrees falls to 5 when we strictly impose the 10% sales truncation. Figure 4 provides a summary of network connections for customers and suppliers in the Compustat linkage data. The left panel shows the distribution of number of links by supplier (out-degree) on a log-log scale, while the right panel shows the distribution of links by customer (in-degree).

Figure 5 reports histograms of weights of customer-supplier sales linkages pooling all supplier-year observations. The distribution for the raw data, in which some suppliers voluntarily report customers below the 10% sales threshold, is on the left. The right panel shows the weight distribution when we strictly impose the 10% sales truncation.

Figure 5: Customer-Supplier Network Linkage Weights



Notes: The figure plots the histogram of Compustat customer-supplier sales weights for the raw data (left panel), and when we strictly imposed the 10% sales threshold in our calibration. Plots pool all firm-years for 1980–2009.

Table 8 reports some key moments of the network data. We calculate cross-sectional correlations for each yearly network realization in the Compustat linkage data and report the time series average of annual correlations. We also report a t-statistic, measured as the ratio of the time series mean of the correlation to its time-series standard deviation. In these calculations, size is defined as annual firm sales. Columns (1)-(5) take the perspective of the suppliers and the connections with their customers. Instead, Columns (6)-(10) focus on costumers' connections to their suppliers. In Panel A we consider all customer-supplier linkages, while in Panel B we only keep those pairs where the customer represents at least 10% of the supplier's sales. Thus, Panel B imposes the truncation also on the firms that voluntarily report more customer data than required.

Panel C adds moments obtained from industry data. Industry input-output data are from the Bureau of Economic Analysis (BEA). Because industry definitions vary quite dramatically over time, we focus on a set of 65 industries we can track consistently over time between 1998 and 2011. This data is informative for evaluating cross-sectional correlations between industry size and network structure since it does not suffer the truncation issue that plagues the Compustat firm-level data. In related work, Ahern and Harford (forthcoming) use the network topography implied by the BEA industry data to show that the properties of these networks have a bearing on the incidence of cross-industry mergers. We do not have return-based volatility measures in the industry data.

The moments reported in Columns 1 and 2 are discussed in Section 2.6, and the moments reported in the other columns are discussed in Section 4.3.3. The numbers reported in the latter section in Table 7 are identical to those reported in Panel B of Table 8. Table 8 additionally provides t-statistics.

B.2 Frequency Decomposition

To study the trend and cycle in the size and volatility moments, we apply the Hodrick-Prescott filter with a smoothing parameter of 50. Figure 6 reports HP-detrended moments. The top-left panel shows firm size dispersion and mean firm volatility based on market capitalization and return volatility. The top right panel reports firm size dispersion and dispersion of firm volatility, also based on market data. The bottom two panels are the counter-parts where size and volatility are based on sales data. The correlations between the cyclical component in average log volatility and size dispersion are 29.2% for the market- and 58.0% for the sales-based measure. The correlations between the cyclical component in volatility dispersion and size

Table 8: Overview of Size, Volatility, and Customer-Supplier Network Structure

	Supplier					Customer				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$w_{i,j,t}$	$\log S_{i,t}$	$\log S_{i,t}$	$H^{out}_{i,t}$	$H^{out}_{i,t}$	$w_{i,j,t}$	$\log S_{i,t}$	$\log S_{i,t}$	$H^{in}_{i,t}$	$H^{in}_{i,t}$
	$S_{j,t}$	$N^{out}_{i,t}$	$H^{out}_{i,t}$	$\log \sigma_{i,t}(r)$	$\log \sigma_{i,t}(s)$	$S_{j,t}$	$N^{in}_{i,t}$	$H^{in}_{i,t}$	$\log \sigma_{i,t}(r)$	$\log \sigma_{i,t}(s)$
Panel A: Compustat Firm Data - Untruncated										
Avg.	0.20	0.03	-0.31	0.15	0.27	0.78	0.37	-0.25	0.12	0.08
$t-stat$	3.61	0.86	-8.69	2.79	3.53	23.44	21.63	-4.34	0.93	0.80
Panel B: Compustat Firm Data - Truncated										
Avg.	0.19	0.01	-0.31	0.15	0.29	0.89	0.37	-0.26	0.13	0.08
$t-stat$	2.03	0.21	-7.08	2.40	3.69	35.92	22.79	-4.00	0.98	0.85
Panel C: BEA Industry Data - Untruncated										
Avg.	0.37	0.61	-0.39	–	0.22	0.41	0.46	0.10	–	0.26
$t-stat$	40.18	28.00	-12.63	–	2.67	44.58	19.33	5.99	–	2.53

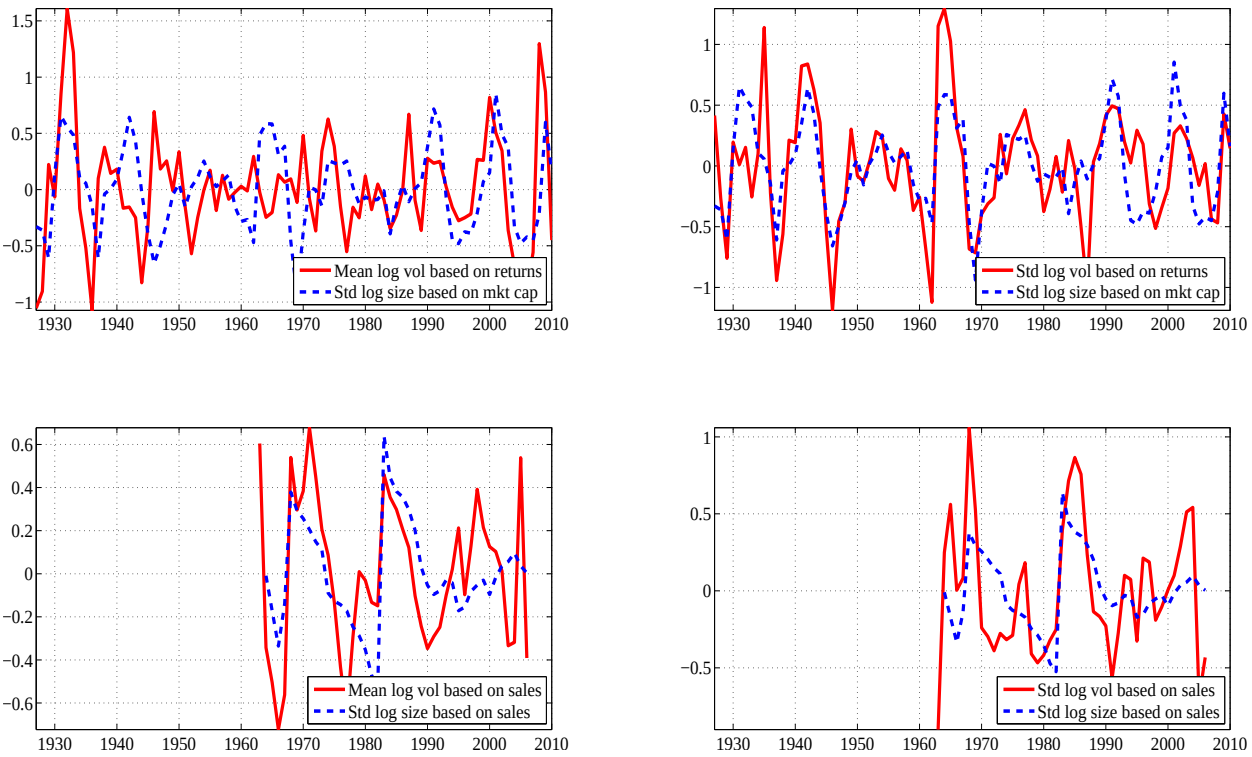
Notes: The table reports cross-sectional correlations between various features of customer-supplier networks with size and volatility. Panels A and B are based on annual firm-level Compustat data for the period 1980-2009. Panel C is based on annual industry-level Bureau of Economic Analysis data for a set of 65 consistently measured industries for the period 1998-2011. The table reports the time-series average as well as the t-statistic, measured as the time-series average divided by the time-series standard deviation estimated over the sample. In Panel A, we use all linkage data in the Compustat data base. In panel B, we impose a 10% truncation, which implies that we discard all customer-supplier pairs that represent less than 10% of supplier sales. The industry data in Panel C are complete and do not suffer from a truncation problem.

dispersion are 65.9% for the market- and 54.4% for the sales-based measure. These results suggest that the correlations between the dispersion in the firm size distribution and moments of the volatility distribution occur at both lower and at cyclical frequencies.

B.3 Granger Causality Tests

Our network model predicts that movements in the size distribution precede changes in the volatility distribution. When the size distribution spreads out (contracts) at time t, the network structure for the subsequent period adjusts, and diversification of growth rate shocks is hindered (enhanced). We use Granger causality tests to formally evaluate whether dispersion in firm sizes predicts the mean and standard deviation of the volatility distribution, after controlling for own lags of the dependent variable. Table 9 presents the results from these tests. We find that firm size dispersion (based on log market equity) has statistically significant predictive power for mean firm volatility (based on returns), and dispersion in volatility. The reverse is not true. After controlling for own lags of size moments, moments of the volatility distribution do not predict the size distribution. Lagged dispersion in log volatility appears to Granger-cause size dispersion, but the coefficient has the wrong sign. The hypothesis has the one-sided alternative that volatility dispersion *positively* predicts size dispersion; thus this negative result leads us to fail to reject the null. This evidence suggests that size dispersion leads the volatility distribution, consistent with the model. This evidence should not be interpreted as support for economic causality, but merely for time-series predictability.

Figure 6: Detrended Size and Volatility Moments



Notes: The figure plots HP-detrended time series moments of size and volatility distributions using smoothing parameter of 50.

B.4 Size and Volatility Moments for Public vs. Private Firms

We investigate whether the properties of firm size and firm volatility are the same for publicly-listed and privately-held firms using three different databases. To the extent that they are, it suggests that our results are not driven by our study of publicly-listed firms.

First, we use the data set provided by Bloom, Floetotto, Jaimovich, and Terry (2012) which allows us to compare measures of firm-level volatility between the universe of publicly-listed firms and the universe of manufacturing establishments. Data on manufacturing establishments are from the Census of Manufactures (CM) and the Annual Survey of Manufactures (ASM) from the U.S. Census Bureau between 1972 and 2009 for establishments with 25 years or more of observations. The data also contain a volatility measure for all public firms with 25 years (300 months) or more in CRSP between 1960 and 2010. Panel A of Table 10 shows the correlation of sales growth volatility and stock return volatility with the volatility of TFP shocks and the volatility of output growth, respectively. IQR of log(TFP) shock (all manufact.) is the cross-sectional interquartile range of the Total Factor Productivity (TFP) "shock" measured at the establishment level, and IQR of output growth (all manufact.) is the interquartile range of plants' sales growth. These two volatility measures are constructed from the Census data. IQR of sales growth (public) is the annual average of the interquartile range of firms' sales growth by quarter for all publicly-traded firms in Compustat between 1962 and 2010. Lastly, IQR of stock returns (public) is the annual average interquartile range of firms' monthly stock returns for all public firms in CRSP between 1960 and 2010. Panel A shows a strong positive correlation between the sales-based and return-based volatility measures for *public* firms, which our study shares with Bloom et al. (2012), and the volatility measures for output growth and TFP shocks, which are constructed from the universe of *private and public* manufacturing establishments.

Table 9: Granger Causality Tests

Dependent Variable			Independent Variables	
		Intercept	$\mu_{\sigma,t-1}$	$\sigma_{S,t-1}$
$\mu_{\sigma,t}$	Coeff	-1.18	0.74	0.07
	t-stat.	*-2.82*	*8.31*	*2.21*
$\sigma_{S,t}$	Coeff	-0.28	-0.11	0.97
	t-stat.	*-0.47*	*-0.88*	*16.01*
		Intercept	$\sigma_{\sigma,t-1}$	$\sigma_{S,t-1}$
$\sigma_{\sigma,t}$	Coeff	-0.02	0.61	0.04
	t-stat.	*-1.48*	*5.86*	*3.12*
$\sigma_{S,t}$	Coeff	0.20	-1.19	1.04
	t-stat.	*1.47*	*-2.25*	*15.88*

Notes: Annual data 1926–2010. The table reports results of Granger causality tests for the ability of log firm size dispersion ($\sigma_{S,t-1}$) to predict the mean ($\mu_{\sigma,t}$) and standard deviation ($\sigma_{\sigma,t}$) of the log volatility distribution. We use the market based volatility measure constructed from stock returns and the market equity measure of size.

Our second data set compares size and volatility moments of Income Statement and Balance Sheet items for the universe of active and inactive firms on Compustat. Inactive Compustat firms are firms that are no longer publicly-listed due to acquisition or merger, bankruptcy, leveraged buyout or change of status to a private company. We use quarterly data from 1962 to 2007, two different measures of size (net sales and pretax income), and we adjust semi-annual and annual reports to quarterly data. There are 25,619 unique firms in the sample, and the number of active and inactive firm-quarter observations is similar. For example, for net sales we have on average 2265 active firms each period and 3530 inactive firms. Size dispersion is computed as the cross section standard deviation of size. To calculate the volatility, we construct the annual growth rate, quarter by quarter, and, then, we define the volatility as the standard deviation of the growth rate over the next 20 quarters. Panel B of Table 10 reports the time series correlation between active and inactive firms' size dispersion, mean volatility and volatility dispersion. The public and private samples display a strong positive correlation in our key moments of the size and volatility distribution.

Third, we look at the evolution of firm size dispersion for public firms versus that for all firms, using the number of employees as the measure firm size. The public data are from Compustat while the firm data for the universe of firms are form the Census' Business Dynamic Statistics. The sample is annual and covers the period 1977-2009. The Census reports employment data in 12 employment bins ranging from $1-4$ employees at the low end to $10,000+$ at the high end. We construct the same bins using the Compustat employment data. We also create a spliced series that divides the 10,000+ bin into 25 sub-bins applying an imputation that replicates the employment distribution in that bin in the Compustat employment data. The evolution of firm size dispersion when considering the entire universe of firms seems similar to the one in Compustat. The correlation between the cross-sectional variance of log size in Compustat and the spliced series is 62%. The correlation between the non-spliced Census measure and the Compustat measure is 65%.

As an aside, we note that at the start of the sample, the Census reports 728 firms with more than 10,000 employees, while Compustat reports 677. So, we have fairly comprehensive coverage at the start of the sample. At the end of the sample, there are 1,975 with 10,000+ firms, only 1,016 of which show up

Table 10: Private versus Public Firms

Panel A: Volatility Measures from Bloom et al. (2012)	
Correlation between	
IQR of sales growth (public) and IQR of log(TFP) shock (all manufact.)	0.49
IQR of sales growth (public) and IQR of output growth (all manufact.)	0.38
IQR of stock returns (public) and IQR of log(TFP) shock (all manufact.)	0.33
IQR of stock returns (public) and IQR of output growth (all manufact.)	0.52

Panel B: Compustat – active versus inactive firms

	Correlation between measures of		
Compustat variable	size disp	mean vol	vol disp
Net Sales	0.74	0.91	0.88
Pretax Income	0.60	0.97	0.55

Notes: Panel A reports time series correlations between various volatility measures constructed from public and private firms by Bloom, Floetotto, Jaimovich, and Terry (2012). In Panel B, we compute time series for firm size dispersion, mean firm volatility, and the dispersion of firm volatility for two non-overlapping samples of active (public) and inactive (private) Compustat firms. We then report the correlation of those statistics between active and inactive firms. The text of Appendix B.4 contains all the details.

in Compustat. There are more large, private firms at the end of the sample. This observation makes the previous exercise, based on active and inactive Compustat firms, extra useful because Compustat captures the large private firms.

In conclusion, the changes in the firm size and volatility distribution that we have documented do not seem specific to the universe of publicly-traded firms. Combined with the evidence for the evidence of the subgroups of publicly-listed firms discussed in Section 3.3, the evidence suggests that the correlation between firm size dispersion and the mean and dispersion of the firm volatility distribution are robust features of the data.

C Calibration Appendix: Downstream Results

We study a version of our model where the direction of shock transmission is reversed from upstream to downstream. A firm's growth rate now depends on its own shock and on the growth rate of its suppliers. We use the exact same calibration as in the benchmark model. If we abstract from truncation, then reversing the direction only changes the network moments reported in Table 7; the moments in the size and volatility tables 5 and 6 are identical. In terms of network moments, the untruncated in-degree and out-degree moments are reversed. However, after truncation (which operates at the level of suppliers), the results look different. Table 11 reports results for the downstream transmission case. This version of the model has more success matching the in-degree moments, but then it does considerably worse matching the out-degree moments in the top panel. Most importantly, this version of the model implies counter-factually that there is no correlation between out-Herfindahls and volatility. In the data, we found that this relation is statistically strong (see Table 8) unlike that between volatility and in-Herfindahl. In addition, Table 3 showed that out-Herfindahl is a robust determinant of firm volatility after controlling for other variables, while the relation between supplier Herfindahls and volatility is not.

Table 11: Downstream Transmission: Network Moments

	All Returns	Top-33% Returns	All Sales	Top-33% Sales	M1 Upstream	Reverse M1 Downstream
			Panel A: Out-degree Moments			
median N^{out}	1.00	1.00	–	–	1.71	3.88
99^{th} % N^{out}	3.38	3.32	–	–	4.66	6.15
median H^{out}	0.05	0.03	–	–	0.60	0.25
99^{th} % H^{out}	0.95	0.54	–	–	1.00	1.00
$Corr(N_t^{out}, S_t)$	0.01	−0.07	–	–	0.34	0.14
$Corr(H_t^{out}, S_t)$	−0.31	−0.08	–	–	−0.71	−0.08
$Corr(H_t^{out}, V_{t+1})$	0.15	0.08	0.29	0.10	0.54	0.05
			Panel B: In-degree Moments			
median N^{in}	1.00	1.12	–	–	1.90	2.34
99^{th} % N^{in}	31.61	16.80	–	–	7.39	62.34
median H^{in}	0.00	0.00	–	–	0.11	0.48
99^{th} % H^{in}	0.28	0.24	–	–	0.85	1.00
$Corr(N_t^{in}, S_t)$	0.37	0.49	–	–	0.49	0.59
$Corr(H_t^{in}, S_t)$	−0.26	−0.20	–	–	0.03	−0.48
$Corr(H_t^{in}, V_{t+1})$	0.13	0.08	0.08	0.08	0.01	0.50

Notes: All reported moments are time-series averages unless explicitly mentioned otherwise. The first and second columns report data for the cross section of firms for which we have customer information from Compustat. The sample is 1980-2009. The variance in Columns 1 and 2 is calculated based on daily stock return data, while the variance in Columns 3 and 4 is based on 20 quarters worth of annual sales growth (in the current and the next four years). Column 5 reports the corresponding moments for the benchmark model (M1). Column 6 shows the benchmark model M1, but with the reverse transmission of shocks from suppliers to customers (Downstream). Log variances in the model are constructed with added estimation noise to make them comparable to the moments in the data.