Disagreement and Discretionary Monetary Policy

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Abstract

This paper identifies a new coordination motive endogenously induced by a central bank's lack of commitment in the presence of information imperfection. We show that when differentially informed economic agents disagree about central bank's inflation incentives, discretion in monetary policy-making induces agents to coordinate by "forecasting the forecasts of others" in order to forecast central bank's policy actions. In particular, the induced coordination mechanism compels the central bank to choose monetary policy that responds to fluctuations in the average belief about its incentive. As a result, discretion has the potential to vastly increase fluctuations in employment and inflation, especially when the disagreement among agents is low. More broadly, our paper makes an argument for the inclusion of information diversity among agents in monetary policy discussions and in the characterization of the inflation dynamics.

*Marvin Goodfriend passed away in December 2019. He was our colleague, mentor, and friend. We dedicate this paper to his memory and his contributions to the field of economics. Goodfriend was a faculty member of the Tepper School of Business at the Carnegie Mellon University and also had a research affiliation with NBER. Pierre Jinghong Liang is from the Tepper School of Business at the Carnegie Mellon University and also has a research affiliation with Shanghai Advanced Institute of Finance and China Academy of Financial Research at Shanghai Jiaotong University. Gaoqing Zhang is from the Carlson School of Management at the University of Minnesota. Pierre Liang gratefully acknowledges the Dean's Summer Research funding of the Tepper School. We thank John McDermott, Ariel Zetlin-Jones and participants at Chicago-Minnesota Theory Conference for valuable comments. Please e-mail comments or suggestions to liangj@andrew.cmu.edu, and zhangg@umn.edu.

1 Introduction

This paper identifies a new coordination motive endogenously induced by a central bank's lack of commitment in the presence of information imperfection. We show that when differentially informed economic agents disagree about the central bank's inflation incentive, discretion in monetary policy-making induces agents to coordinate by "forecasting the forecasts of others" in order to forecast central bank's policy actions. We abstract from inherent interdependencies that have been studied in the past to isolate the cause and the effect of our coordination motive.[1] In particular, our discretion-disagreement coordination mechanism compels the central bank to choose monetary policy that responds to fluctuations in the average belief about its inflation incentive which, in turn, is what forces agents to coordinate by forecasting the forecasts of others. As a result, discretion has the potential to vastly increase fluctuations in employment and inflation, especially when the disagreement among agents is low. More broadly, our paper makes an argument for the inclusion of information diversity among agents in monetary policy discussions and in the characterization of the inflation dynamics.

We adopt two information imperfections involving the central bank's inflation incentive: i) disagreement among individual agents, and ii) the average forecast error of all agents. Specifically, agents forecast future inflation incentives imperfectly and asymmetrically with a private signal that contains a common noise and an idiosyncratic noise. The common noise yields a stochastic average forecast error and the volatility of the idiosyncratic noise governs dispersion of the individual forecasts around the average forecast. Surprisingly, we find that in equilibrium under discretion holding fixed average forecast accuracy, **more agreement** among agents destabilizes employment and inflation. Equally surprising, we find that **more accurate** average forecasts also destabilize employment and inflation in equilibrium under discretion when agents are sufficiently in agreement with each other. In effect, more agreement among agents coordinates forecasts more tightly and magnifies the effect of the common noise on employment and inflation via the central bank's more aggressive reaction to the average forecast. The magnified common noise constitutes an information-based source of macroeconomic instability which has not been identified previously.

[1]Coordination motives may also arise from inherent interdependencies among actors, either through technology linkage (Angeletos and Pavan 2004), information extraction (Townsend 1983), monopolistic competition (Woodford 2001), trading (Angeletos and La'O 2013) or beauty-contest preference (Morris and Shin 2002).

The coordination problem we identify exists only under discretionary monetary policy. The problem goes away when the central bank follows a credible rule, even in the presence of imperfect information. Specifically, with commitment, the central bank can unilaterally and uniformly anchor each individual firm's expectation by credibly specifying both current and future policies. As a result, the central bank's equilibrium policy actions would be based upon the pre-determined decision rule that is known to all firms. Although this rule may depend on future shocks to the inflation incentive that are imperfectly known to firms, firms can simply estimate these shocks by constructing first-order beliefs, without necessarily constructing (higher-order) beliefs about others' beliefs.

Without commitment, the central bank's current action loses control of the average current expectation of its future policy actions and worse, it must react to its assessment of the average expectation. Therefore, when an individual agent forecasts future policy actions, the agent must forecast the average expectation which depends on other agents' forecasts. As a result, the average (first-order) belief of future policy actions now depends on the average forecast of other agents' forecasts, or the average second-order belief. Similarly, the average second-order belief would, in turn, depends on third-order beliefs, and so on.

In equilibrium under discretion, the aggregate variables such as output and inflation are functions of the average forecast of future monetary policy actions; the average forecast is determined by a hierarchy of higher-order beliefs which, in turn, depends on the properties of the average forecast and the degree of disagreement. Recognizing the complexity of the problem, we characterize the effects of higher-order beliefs on aggregate variables in a linear manner within a New Keynesian (New Synthesis) macroeconomic model, solved in closed-form with a class of normally distributed signals. With this specification, equilibrium inflation and aggregate output respond to the average forecast-error linearly. The degree of disagreement, among other model parameters, affects the equilibrium sensitivity of inflation and output to the average forecast-error because the degree of disagreement affects the aggressiveness with which the central bank, under discretion, is forced to respond to the average forecast-error.

Given the equilibrium monetary policy, inflation and output become more volatile due to the addition of the shock on the average forecast. The induced coordination–the cause of heightened macro fluctuations–makes the problem especially pronounced due to a "multiplier" effect of the average forecast error. This is because the same private signal is used for each level (ladder) in the individual higher-order belief hierarchy; averaging across all individual only eliminates the idiosyncratic noise but not the common noise. Thus, the common noise is retained at each higher-order level of the average belief, magnifying the noise contained in the average forecast in equilibrium. When a discretionary central bank reacts to the average forecast, the magnified average forecast error enters into aggregate inflation and output, generating volatility due to the information imperfection beyond those "real" shocks commonly studied such as cost-push or demand shocks.

Facing such a pronounced problem caused by information imperfection, the conventional wisdom would suggest that reducing the volatilities of the information shocks would be desirable. We find that this intuition does not hold generally. Holding fixed the average forecast-error volatility, a higher degree of disagreement among agents makes them less coordinated as each firm relies less on its signal when forming expectation about future policy actions. The central bank, in turn, becomes less responsive to the average forecast. Thus, fluctuations in employment and inflation due to information shocks are lower with a higher degree of disagreement. So narrowing the degree of disagreement would introduce more economic fluctuations and destabilizes inflation and output.

On the other hand, holding the degree of disagreement fixed, an increase in the precision of the average forecast creates a trade-off between a direct reduction in the size of common noise and an indirect increase in sensitivity to the noise. Specifically, an increase in the precision directly reduces the common noise in the central bank's equilibrium monetary policy, leading to less volatility in the output and inflation series holding fixed the central bank's reaction sensitivity. However, when the precision of average forecast increases, all agents are better informed about future inflation and adjust current output; anticipating such adjustment, the discretionary central bank reacts by increasing the sensitivity of its current policy action to the average forecast, including the error therein, adding more volatility to equilibrium inflation and output. When the degree of disagreement is high (low), the trade-off favors the direct (indirect) effect. Consequently, reducing the average forecast-error volatility stabilizes employment and inflation in equilibrium only when the degree of disagreement is high enough.

Our model specification borrows key elements from two distinct literatures: (1) macroeconomic research focusing on monetary policy and (2) information economics research focusing on information structure. Based on the large literature on monetary policy research, we deploy the structural equations summarizing key insights from the New Keynesian model as described in the survey paper by Clarida, Gali and Gertler (1999). Our model shares with Cukierman and Meltzer (1986), an early work on central bank opacity, a key feature that the central bank inflation incentive is stochastic and perpetually obscured. From the information economic research, we draw from research on information structure by economists as well as accounting researchers. The key element of our information structure–correlated private signals–has been used in the studies of financial markets (Holthausen and Verrecchia 1990) and recently have been studied in coordination settings with inherent interdependencies (Myatt and Wallace 2012, Liang and Zhang 2019).

To appreciate the connection this paper makes, consider the two debates in monetary policy that have received much academic, practical, and policy attention. The rules-versus-discretion debate has a long and varied standing. According to McCallum (1999, page 1485), a "major reorientation" dates back to Barro and Gordon (1983) "built upon the insights of Kydland and Prescott (1977)." As is well-known, the main insight identified by this literature is that discretionary polices suffer from the time-inconsistency problem: the market participants' rational expectation renders discretionary policies, designed by a benevolent central bank, ineffective and, worse, generating unnecessary inflation and economy fluctuations. The transparency-opacity debate in monetary policy can be traced back to Cukierman and Meltzer (1986) and Goodfriend (1986). This debate explicitly considers the potential information asymmetry between the central bank and the market participants (see, e.g., the survey by Geraats 2002). For example, collectively the public may perceive a lack of access into the workings of the central bank in promulgating monetary policy, leading to a perceived opacity of central bank (e.g., Winkler 2002). Our paper bridges the two debates by identifying the link in between. In this light, our paper is related to Morris and Shin (2005) who also point to the connection between central bank discretion and transparency.[2] Interestingly, Morris and Shin (2005) also stress the preeminent role of managing expectations in linking the debate of central bank transparency and monetary policy to the extent that the central bank manipulates market expectations via communication and, at the same time, extracts information from market prices to guide monetary policy. In a sense, our paper complements the insight of Morris and Shin

[2] Morris and Shin (2005) articulate a general point about this link from a political economy perspective: "In light of the considerable discretion enjoyed by independent central banks, the standards of accountability that they must meet are perhaps even higher than for most other public institutions. Transparency allows for democratic scrutiny of the central bank and hence is an important precondition for central bank accountability." (Morris and Shin 2005, page 1).

(2005) by outlining an alternative mechanism through which market expectations about the central bank's policy target interact with the monetary policy the central bank sets at its discretion.

Students of central banks have long noted the importance of the disagreement among individuals. For instance, Brunner (1981) studies the disagreement among individual agents' subjective perceptions of the monetary policy. King (1982, 1983) and Dotsey and King (1986) study the informational implication to monetary policy when differentially informed agents extract endogenous information from prices. Outside the two debates on central bank discretion and transparency, the pioneering idea by Phelps (1983) have stressed the lack of common knowledge in explaining the aggregate economic dynamics. The initial work by Townsend (1983) analytically formulated the idea of forecasting the forecasts of others. Woodford (2001), among other recent works, relies on finite information-processing capacity (Sims 2003) to show that informational disagreement among individuals leads them to construct beliefs about others' beliefs, or higher-order beliefs, within the contemporary framework of macro-models. More recently, a growing literature (e.g., Angeletos and Lian 2018; Angeletos and La'O 2020; Angeletos and Huo 2021) has begun to examine the role of dispersed information and higher-order uncertainty in a large economy.[3] However, they focus on dispersed information about the state of the economy (as opposed to dispersed information about the discretionary policy target of the central bank which we focus on). Our paper complements this literature by adding a new coordination motive driven by central bank discretion and transparency.

The rest of the paper proceeds as follows. Section 2 lays out the basic macroeconomic framework and the key elements of our information assumptions. Section 3 analyzes the resulting model and constructs the central higher-order-belief arguments. Section 4 analyzes a parameterized version of the model. Section 5 concludes.

2 Model Setup

2.1 A Simple Macroeconomic Framework

The economy is populated with a central bank that takes the nominal interest rate as the instrument of monetary policy, a representative household and a continuum of firms, indexed by $[0, 1]$. Rather

[3] A comprehensive review of this literature is beyond the scope of our paper. We refer interested readers to the excellent survey by Angeletos and Lian (2016, Section 8).

than deriving the optimal conditions for the household and firms, we describe the operation of the economy by a set of structural equations that can be derived from log-linearizing optimal consuming and profit maximizing conditions (as in Galí 2008). Let y_t and y_t^p denote the logs of the aggregate economy output and the potential level of the output. The potential output is the level of output that would arise if wages and prices were fully flexible but may be lower than efficient level due to existing frictions such as monopolistic competition, taxes and subsidies. Define the output gap x_t as the difference between y_t and y_t^p:

$$x_t \equiv y_t - y_t^p. \tag{1}$$

In addition, let π_t be the inflation rate from period $t-1$ to t.

First, there is a new Keynesian Phillips curve that links inflation π_t to output gap x_t, generated by the firms in the economy:

$$\pi_t = \lambda x_t + \beta \bar{E}_t^F \pi_{t+1} + u_t, \tag{2}$$

where $\bar{E}_t^F[\cdot]$ denotes the average belief of the firms, i.e., $\bar{E}_t^F[\cdot] = \int_0^1 E_t\left[\cdot|I_t^i\right] di$, with firm i's information set I_t^i.[4] The shock u_t follows

$$u_t = \rho_u u_{t-1} + \hat{u}_t, \tag{3}$$

where $\rho_u \in [0,1)$ and $\hat{u}_t$ are i.i.d. random variables with zero mean and variances σ_u^2. The Phillips curve can be derived by profit maximizing conditions by firms that compete with each other monopolistically and face nominal price rigidities (Calvo 1983; Yun 1996; Woodford 2008). The key feature of the Phillips curve is that the average expected future inflation $\bar{E}_t^F[\cdot]$ enters, which creates a role for the beliefs of the firms in affecting equilibrium inflation and output levels. This role, in turn, influences the central bank's monetary policy in equilibrium, making it partially self-fulling.

Second, there is a dynamic "IS" equation that describes the relation between real interest rate and output gap generated by the representative household in the economy:

$$x_t = -\phi r_t + E_t^H x_{t+1} + g_t, \tag{4}$$

[4]It can be shown that this version of New Keynesian Phillips curve (2) can be derived from a suitable micro-foundation where each firms is endowed with different information set I_t^i.

where r_t is the real interest rate (from period t to period $t+1$) and $E_t^H[\cdot]$ denotes the expectation by the representative household. g_t is a shock that follows,

$$g_t = \rho_g g_{t-1} + \hat{g}_t, \tag{5}$$

where $\rho_g \in [0,1)$ and $\hat{g}_t$ are i.i.d. random variables with zero mean and variances σ_g^2. The IS equation can be derived from log-linearizing the Euler equation of the representative household.

Third, a Fisher equation links the nominal interest rate to the ex ante real interest rate and the representative household's expected inflation. Let i_t be the nominal interest rate from period t to $t+1$.

$$i_t = r_t + E_t^H \pi_{t+1}. \tag{6}$$

Replacing r_t in the IS equation with $r_t = i_t - E_t \pi_{t+1}$ in the Fisher equation gives a modified IS equation,

$$x_t = -\phi \left(i_t - E_t^H \pi_{t+1} \right) + E_t^H x_{t+1} + g_t. \tag{7}$$

The central bank in period t minimizes deviations of aggregate output gap and inflation from their respective targets:

$$\frac{1}{2} E_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\alpha \left(x_{t+\tau} - k_{t+\tau} \right)^2 + \pi_{t+\tau}^2 \right] \right\}, \tag{8}$$

subject to the Phillips curve (2) and the IS curve (7), where α is the relative weight on output deviation. We interpret the loss-function as endowing the central bank a dual mandate: a zero-inflation target and output gap target. We assume that the target for the output gap is k_t. Adapting Barro and Gordon (1983), k_t represents the extent the central bank intends to raise actual output above potential (toward efficient output). For example, $k_t = 0$ implies that the central bank is satisfied with aggregate output at potential output level (but below efficient level).[5] When $k_t > 0$, the central bank has an incentive to target actual output above the potential level, generating an incentive to inflate. As is typically done, we will use k_t to represent both higher output target than potential and inflation incentives interchangeably.

[5] One may interpret a zero or low k_t as either the central bank truely believes that potential output is very close to efficient output or that potential output is far below to efficient output but a discretionary central bank recognizes its own limitation due to time-inconsistence program and chooses to tolerate the inefficiencies and thus lower inflation incentive.

2.2 Information Environment

We first describe in detail the information environment followed by description of the resulting updating mechanism used by each firm when forecasting central bank's future inflation incentive. Every period, two standard macro shocks $\{u_t, g_t\}$ and the inflation incentive shock k_t are contemporaneously observable to all firms, the representative household and the central bank. The incentive shock k_t follows,

$$k_t - \bar{k} = \rho_k \left(k_{t-1} - \bar{k}\right) + \nu_t, \tag{9}$$

where $\rho_k \in [0, 1)$ and $\nu_t \sim N(0, \frac{1}{q})$. As a result, $k_t \sim N\left(\bar{k}, \frac{1}{q_k}\right)$, where $\bar{k} > 0$ and $q_k = q\left(1 - \rho_k^2\right)$.

In addition, each individual firm receives a *private* foreknowledge about future inflation incentive. Specifically, at time t firm-i receives a signal s^i_{t+j} informative about the j-period ahead inflation incentive shock k_{t+j}. The signal is modeled as

$$s^i_{t+j} = k_{t+j} + \eta_{t+j} + \varepsilon^i_{t+j}, \tag{10}$$

where $\eta_{t+j} \sim N\left(0, \frac{1}{m}\right)$ is common across firms and $\varepsilon^i_{t+j} \sim N\left(0, \frac{1}{n}\right)$ is idiosyncratic among firms.[6] Each private signal contains two shocks representing the two information imperfections. First, the average signal is an forecast of the future inflation incentive but with error, measured by η_{t+j}. Denote $\bar{s}_{t+j} = \int_0^1 s^i_{t+j} di$ the average signal of all firms, we have

$$\text{Average Forecast Error: } \bar{s}_{t+j} - k_{t+j} = \eta_{t+j} \quad \text{and} \quad Var(\eta_{t+j}) = \frac{1}{m}. \tag{11}$$

The volatility of average forecast error is measured by its variance $\frac{1}{m}$. The larger m is, the more precise $\bar{s}_t$ about the central bank's incentive k_t. Second, the idiosyncratic shock in each signal generates disagreement among agents:

$$\text{Disagreement: } s^i_{t+j} - \bar{s}_{t+j} = \varepsilon^i_{t+j} \qquad \text{and} \quad Var(\varepsilon^i_{t+j}) = \frac{1}{n}, \tag{12}$$

at any time t and firm i. The degree of disagreement among firms is measured by $\frac{1}{n}$, the variance

[6] s^i_{t+j} can be interpreted as a sufficient signal summarizing any new information regarding k_{t+j} that arrives in period t. It can be interpreted as from (unmodeled) private information acquisition, central bank disclosure, or learning from observing noisy signals of past inflation and output gap, etc. (Cukierman and Meltzer 1986).

of ε_t^i. The larger n is, the smaller the disagreement across firms. Notice that our specification of the information structure allows us to capture the precision of average forecast error independently from the disagreement among firms. Adopting an information structure which models disagreement independently from average forecast-error is critical for our model. If each private signal only contains idiosyncratic noise, average forecast would be perfect by assumption, no matter what other imperfect public information are available.[7] In this regard, our modeling choice here is motivated by insights generated by the decades of theoretical research on accounting information structure.[8]

Every period, firm i uses information set I_t^i, to forecast relevant future shocks in order to form beliefs about future inflation. We assume a firm i's relevant information set is

$$I_t^i = \left\{ \{u_\tau\}_{\tau=0}^{t}, \{g_\tau\}_{\tau=0}^{t}, \{k_\tau\}_{\tau=0}^{t}, \{s_\tau^i\}_{\tau=0}^{t+j} \right\}, \tag{13}$$

which includes all the past observations of k_τ up to period t and all the past acquired signals s_τ^i up to period $t+j$.[9] Using I_t^i to update beliefs about future k follows the Bayes' Rule.[10]

Every period t, the central bank's information set is $I_t^{CB} = \{\{u_\tau\}_{\tau=0}^{t}, \{g_\tau\}_{\tau=0}^{t}, \{k_\tau\}_{\tau=0}^{t}, \{\bar{s}_\tau\}_{\tau=0}^{t+j}\}$ and chooses policy instrument i_t to achieve its objective.[11] This assumption supported by the ob-

[7] In effect, making this seemingly common and innocuous information assumption would inadvertently build in a collective rationality that precludes analysis of the kind of coordination mechanism that we study here.

[8] Starting in the late 1960s and early 1970s, accounting researchers began linking accounting concepts to information economics concepts (see AAA monographs by Feltham 1972 and Mock 1976). The agenda is to build on the traditional approach under a purely measurement perspective and to tie the accounting measurement concepts to economic trade-off in decision making under uncertainty. A seminal contribution is by Ijiri and Jaedicke (1966) who framed objectivity within statistical sampling setting as inter-personal agreement and relate it to reliability. Ijiri and Jaedicke introduced two properties of accounting measurement structure. One is the distance between the true state and the average measurements, which we define as Average Forecast Error in our paper. The other one is the distance between the average measurements and measurements by different measurers, which we define as disagreement.

[9] The sources of information available for each firm are exogenously given. We view I_t^i as sufficient statistics for firm i to forecast future inflations at time t. Endogenous information sources may include potentially noisy observations of output and prices such as norminal interest rate and inflation series. We abstract away from these endogenous sources to focus on the role of disagreement, however it is generated, on macro variables.

[10] The computations of first-, second-, or higher-order expectations can be very simple or quite complex depending on parameters. Consider a simple case of $j = 2$ and $\rho_k = 0$, in order to form a first-order belief about next period's inflation incentive k_{t+1}, the firm i would only use s_{t+1}^i to compute its individual conditional expectation of k_{t+1} (as all other signals are uselss due to the independence assumptions). For a computation of the (higher-order) beliefs, see the discussion of Proposition 2 in Section 4. When ρ_k is not zero, these expectation computation involves more terms as more signals are now informative about future central bank incentives. For example, with a non-zero ρ_k, $\{k_t, s_{t+1}^i, s_{t+2}^i, ...s_{t+j}^i\}$ are all informative about k_{t+1}. See Proposition 2 in Section 4 for a detailed account of such first-, second-, and higher-order expectations when the inflation incentives k_t's are serially correlated.

[11] Technically in a simultaneous-move game, a Nash equilibrium only requires the central bank to choose a best-response to average expectations, not necessarily to observe the actual average expectation. Therefore, the observability of average signals by the central bank is inconsequential.

servation that a central bank is typically endowed with more information than an individual firm. The representative household's information set is $I_t^H = \left\{ \{u_\tau\}_{\tau=0}^t, \{g_\tau\}_{\tau=0}^t, \{k_\tau\}_{\tau=0}^t \right\}$ and chooses intertemporal consumption with rational expectation. As we will show later, the Phillips curve that effectively determines the equilibrium inflation and output gap does not include the expectation of the representative household $E_t^H[\cdot]$. As a result, $E_t^H[\cdot]$ (thus his information set) only affects nominal interest rate through the dynamic "IS" curve but does not affect the equilibrium inflation and output.

3 Preliminary Policy Analysis with Disagreement and Discretion

We assume that the central bank conducts a discretionary monetary policy each period. In a typical period t, the firms and the central bank simultaneously decides their actions. Specifically, the central bank chooses the nominal interest i_t given its information set I_t^{CB} while each firm forms an expectation (forecasts) about the inflation rate in the next period, given its information set I_t^i and its conjecture about the central bank's future actions. In short, the players plays a simultaneous-move game according to their best-response given their own information set. This section provides the preliminary analysis needed to construct the closed-form equilibrium outcome in Section 4.

3.1 First Order Condition for the Central Bank

Since the central bank cannot commit, it only chooses the current nominal interest rate i_t (but not future rates) that solves the following optimization program:

$$\begin{aligned} \min_{i_t} \quad & \frac{1}{2} E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left[\alpha \left(x_{t+\tau} - k_{t+\tau} \right)^2 + \pi_{t+\tau}^2 \right] \right\}, \\ s.t. \quad & x_t = -\phi \left[i_t - E_t^H \pi_{t+1} \right] + E_t^H x_{t+1} + g_t, \\ & \pi_t = \lambda x_t + \beta \bar{E}_t^F \pi_{t+1} + u_t. \end{aligned} \tag{14}$$

Following Clarida, Gali and Gertler (1999), we solve the optimization program in two stages: first, we solve for the pair of (x_t, π_t) that maximizes the objective given the Phillips curve (2); second, we use the IS curve (7) to determine the nominal interest rate i_t that supports the optimal pair of (x_t, π_t). Throughout the paper, since we are mostly interested in the equilibrium properties of inflation and output, we will focus on analyzing the first stage. Accordingly, we will omit the superscript F in expectation notation $\bar{E}_t^F$ and use $\bar{E}_t$ to represent average expectation by the firms in the rest of the paper to simplify exposition. In the first stage, notice that since the central bank, under discretion, cannot credibly change the firms' beliefs about it future actions, it takes the firms' expectations as given. As a result, the optimization problem for the central bank can be simplified into:

$$\begin{aligned} &\min_{\{x_t, \pi_t\}} \frac{1}{2}\left[\alpha\left(x_t - k_t\right)^2 + \pi_t^2\right] + F_t, \\ s.t. \qquad &\pi_t = \lambda x_t + f_t, \end{aligned} \tag{15}$$

where $f_t = \beta \bar{E}_t \pi_{t+1} + u_t$, and $F_t = \frac{1}{2} E_t \left\{\sum_{\tau=1}^{\infty} \beta^i \left[\alpha\left(x_{t+\tau} - k_{t+\tau}\right)^2 + \pi_{t+\tau}^2\right]\right\}$.[12] The first-order condition on x_t gives:

$$x_t = -\frac{\lambda}{\alpha}\pi_t + k_t. \tag{16}$$

The first-stage solution reveals that the central bank must choose its policy instrument (in the second-stage) to respect equation (16). Holding k_t constant, a central bank seeing a positive (cost-push) shock u_t that pushes current inflation π_t higher via the Phillips curve would choose a policy to reduce current output, thus lowering the output gap x_t. Equation (16) also shows that the central bank is tempted to raise the output gap by k_t, holding the (cost-push) shock u_t constant. The higher the k_t, the higher the central bank's temptation to push up the output gap.

Substituting the first-order condition (16) into the Phillips curve (2) reveals inflation expectation dynamics generated by central bank's best-response:

$$\pi_t = \frac{\alpha\lambda}{\alpha + \lambda^2} k_t + \frac{\alpha\beta}{\alpha + \lambda^2} \bar{E}_t\left[\pi_{t+1}\right] + \frac{\alpha}{\alpha + \lambda^2} u_t. \tag{17}$$

Equation (17) suggests that the central bank must respond to changes in average expectations

[12] To focus our attention to the role of higher-order beliefs on monetary policy, we ignore alternative equilibria involving reputation (using, e.g., grim-trigger strategies) which could support a more efficient outcome (see text by Mailath and Samuelson 2006). See footnote 26 on page 1671 of Clarida Gali and Gertler (1999) for a background and explanations.

$\bar{E}_t[\pi_{t+1}]$ in determining the inflation. The higher the expected future inflation $\bar{E}_t[\pi_{t+1}]$, the higher the actual current inflation π_t. In this sense, (17) captures the self-fulfilling nature in monetary policy making. The coefficient before $\bar{E}_t[\pi_{t+1}]$, $\frac{\alpha\beta}{\alpha+\lambda^2} \in (0,1)$, thus measures how responsive the actual current inflation is to the expected future inflation.

3.2 Forward Recursive Solutions of Phillips Curve under Disagreement and Discretion

We solve for π_t through forward-looking iteration. Iterating (17) once gives

$$\begin{aligned} \pi_t &= \frac{\alpha}{\alpha+\lambda^2}u_t + \frac{\alpha\beta}{\alpha+\lambda^2}\frac{\alpha}{\alpha+\lambda^2}\bar{E}_t[u_{t+1}] + \frac{\alpha\lambda}{\alpha+\lambda^2}k_t \\ &\quad + \frac{\alpha\beta}{\alpha+\lambda^2}\frac{\alpha\lambda}{\alpha+\lambda^2}\bar{E}_t[k_{t+1}] + \left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^2 \bar{E}_t\bar{E}_{t+1}[\pi_{t+2}]. \end{aligned} \tag{18}$$

The key observation is that, in contrast to symmetric information case (i.e., no disagreement), the law of iterated expectation does not hold for average beliefs by differentially informed firms (Morris and Shin 2002)[13]. That is,

$$\bar{E}_t\bar{E}_{t+1}[\cdot] \neq \bar{E}_t[\cdot]. \tag{19}$$

In fact, $\bar{E}_t\bar{E}_{t+1}[\cdot]$ corresponds to the second-order average beliefs of the firms, i.e., the firms' beliefs about the others' beliefs, which may differs substantially from the first-order average beliefs $\bar{E}_t[\cdot]$ when firms are differentially informed about central bank incentives. Similarly, a third-order belief term would show up when equation (17) is iterated twice, and so on. Because of the failure of the law of iterated expectation, we must characterize the entire hierarchy of higher-order beliefs, all of which depend on the firms' current information set (I_t^i) and affect the equilibrium monetary policies. To simplify notations, we denote the l-th order beliefs as $\bar{E}_t^l[\cdot]$, where

$$\bar{E}_t^l[\cdot] \equiv \bar{E}_t\bar{E}_{t+1}...\bar{E}_{t+l-1}[\cdot]. \tag{20}$$

We find that the iteration of (17) converges and gives π_t as a function of the higher-order beliefs, as summarized in the proposition below.

[13] We will verify this point once we specify information structure for firms in the next section.

Proposition 1 *In equilibrium, the inflation rate π_t depends on the sum of the higher-order beliefs about $\{k_{t+i}\}_{i=0}^{i=\infty}$, i.e.,*

$$\pi_t = \frac{\alpha\lambda}{\alpha+\lambda^2}k_t + \frac{\alpha u_t}{\alpha\left(1-\beta\rho_u\right)+\lambda^2} + \left\{\sum_{l=1}^{\infty}\left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^l \frac{\alpha\lambda}{\alpha+\lambda^2}\bar{E}_t^l\left[k_{t+l}\right]\right\}. \tag{21}$$

Before we consider the specific linear-normal information structure laid out earlier, we note a key difference between our setting and other settings that also study higher-order beliefs. In many other settings (e.g., Morris and Shin 2002; Angeletos and Pavan 2004, 2007), the hierarchy of higher-order beliefs plays important roles because these settings explicitly assume a coordination problem (or inherent strategic interdependencies) among individual players and thereby makes it necessary for each player to forecast others' beliefs in order to better coordinate. Our model does not rely on assuming such explicit coordination motives.[14] The individual firms in our model are strategically independent of each other and only striving to form their own best forecast of the future inflation rate. In fact, the firms would not be concerned with others' forecasts/beliefs if the central bank were able to commit to future policy ex ante. This is because, in that situation, the central bank's equilibrium actions would simply be a decision rule.[15] In equilibrium, the firms would take the committed decision rule as given–instead of having to make a conjecture–and thus their forecasting problem would reduce to estimating the unobservable parameters in the decision rule, for instance, the central bank's inflation incentives $\{k_{t+\tau}\}_{\tau=0}^{\infty}$. As a result, each firm would only need to form a first-order belief about the unobservable variables and the other higher-order-beliefs terms would become irrelevant. In other words, the central bank's commitment eliminates the need for forecasting the forecasts of others.

In our setting, instead of explicitly assumed, the coordination motive among the firms is "induced" by the central bank's discretion, in combination with rational expectation by the firms and the self-fulfilling feature embedded in the Phillips curve. When the central bank cannot commit to

[14] Townsend (1983) is notable study on forecasting others' forecasts that does not rely on explicit coordination incentives. Different from our paper, forecasting others' forecasts in Townsend's setting is driven by information spillover of other firms investment choices. That is, observing other firms' investment choices helps the forecasting firm learn more about its own investment environment.

[15] Notice that this does not imply that the central bank is not rational in choosing its actions. When it could commit, the central bank would correctly conjecture the firms' expectations in equilibrium and choose its actions accordingly given the Phillips curve. Therefore, its actions would depend on the firms' equilibrium expectations but not the actual expectations.

future decision rules, all parties know that it will adjust its actions in response to the firms' aggregate expectations, $\bar{E}_t[\pi_{t+1}]$ every period. In mechanical terms, it will choose the pair of $\{x_t, \pi_t\}$ for a given $\bar{E}_t[\pi_{t+1}]$, the aggregate expectation of its own future action, based on the Phillips curve relation. From an individual firm's perspective, since others' forecasts collectively affect the central bank's monetary actions which, in turn, affect the very inflation rate it wants to forecast to begin with, it must also forecast the forecasts of others. In this process, rationality dictates that it must form beliefs about others' beliefs about $\{k_{t+\tau}\}_{\tau=0}^{\infty}$, others' beliefs about others' beliefs and even higher-order beliefs. These beliefs in turn determine individual forecasts of all firms, which collectively influence the equilibrium inflation through the self-fulfilling feature embedded in the modified Phillips curve (17). Notice that from equation (21), the relative importance of higher-order beliefs is determined by $\frac{\alpha\beta}{\alpha+\lambda^2}$, the responsiveness of the actual inflation to the expected future inflation. If $\frac{\alpha\beta}{\alpha+\lambda^2} = 0$, the equilibrium inflation becomes independent of the aggregate expectation, making it unnecessary for each firm to forecast others' forecasts. As a result, all the higher-order-belief terms vanish.

3.3 A Closed-form Forward Recursive Solution

As a matter of exposition and practice, we believe allowing two-period ahead foreknowledge (i.e., setting $j = 2$) is sufficient, in part, because it allows a closed-form solution to the full equilibrium (the derivations for the cases with $j > 2$ are similar but less analytically tractable). Specifically, at any period t, a firm's information set is $I_t^i = \left\{\{u_\tau\}_{\tau=0}^{t}, \{g_\tau\}_{\tau=0}^{t}, \{k_\tau\}_{\tau=0}^{t}, \left\{s_\tau^i\right\}_{\tau=0}^{t+2}\right\}$. To proceed, we first remove redundant elements in the firm's information set. First, at each period t, observe that $\{k_\tau\}_{\tau=0}^{t}$ are commonly known and are sufficient statistics for signals, $\left\{s_\tau^i\right\}_{\tau=0}^{t}$, so the only useful signals are $\left\{s_{t+1}^i, s_{t+2}^i\right\}$ when forecasting future k's. Second, since k_t follows an AR(1) process, k_t is a sufficient statistics for all the past $\{k_\tau\}_{\tau=0}^{t-1}$. To sum, a firm's information set can be simplified into $I_t^i = \left\{k_t, s_{t+1}^i, s_{t+2}^i\right\}$ for the purpose of forecasting future k's.

The following proposition provides the closed-form solutions to the higher-order beliefs terms:

Proposition 2 *When $j = 2$, the l-th order average beliefs becomes*

$$\bar{E}_t^l[k_{t+l}] = \bar{k} + \rho_k^{l-1}\left\{[1 - w(l)]\,\bar{E}_t\left[k_{t+1} - \bar{k}|s_{t+1}^i, k_t\right] + w(l)\,\frac{\bar{s}_{t+2} - \bar{k}}{\rho_k}\right\}, \tag{22}$$

where $\bar{E}_t\left[k_{t+1}-\bar{k}|s^i_{t+1},k_t\right]=\frac{q}{q+\frac{mn}{m+n}}\rho_k\left(k_t-\bar{k}\right)+\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\bar{s}_{t+1}-\bar{k}\right)$ *and* $w(l)$ *is a constant given in the appendix.*

Proposition 2 suggests that $\bar{E}_t\left[k_{t+1}|s^i_{t+1},k_t\right]$ and $\bar{s}_{t+2}$ are the two sufficient statistics for period-t firms to forecast the average higher-order beliefs about the central bank's future inflation incentive. To further illustrate the construction of the higher-order-belief hierarchy, consider first a special case in which the central bank's inflation incentive k_t are serially uncorrelated ($\rho_k=0$). In this case, firms share a common prior on $k_t\sim N\left(\bar{k},\frac{1}{q}\right)$. In addition, when forecasting k_{t+l}, the only useful signals are the prior $\bar{k}$ and s^i_{t+l}, and all the other signals, $\left\{s^i_\tau\right\}^t_{\tau\neq t+l}$, are not useful, since k_t is serially uncorrelated. In this case, the first-order belief $\bar{E}_t\left[k_{t+1}\right]$ is a weighted average of the prior and the average signal $\bar{s}_{t+1}$, with the weights simply the ones under Bayesian updating and similarly for the first-order belief $\bar{E}_t\left[k_{t+2}\right]$ i.e.,

$$\bar{E}_t\left[k_{t+1}\right]=\bar{k}+\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(\bar{s}_{t+1}-\bar{k}\right),\qquad(23)$$

$$\bar{E}_t\left[k_{t+2}\right]=\bar{k}+\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(\bar{s}_{t+2}-\bar{k}\right).\qquad(24)$$

To form the average second-order belief, $\bar{E}^2_t\left[k_{t+2}\right]$, first consider an individual firm i's expectation of next-period's average belief:

$$\begin{aligned}E^i_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right]&=\bar{k}+\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(E^i_t\left[\bar{s}_{t+2}\right]-\bar{k}\right)\qquad(25)\\&=\bar{k}+\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(\bar{k}+\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(s^i_{t+2}-\bar{k}\right)-\bar{k}\right)\\&=\bar{k}+\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(s^i_{t+2}-\bar{k}\right),\end{aligned}$$

and aggregating all firms' expectations, the average second-order belief becomes,[16]

$$\bar{E}^2_t\left[k_{t+2}\right]\equiv\bar{E}_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right]=\bar{k}+\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}\left(\bar{s}_{t+2}-\bar{k}\right).\qquad(26)$$

Notice that in forming the average second-order belief $\bar{E}^2_t\left[k_{t+2}\right]$, the average signal $\bar{s}_{t+2}$ is assigned

[16] One can also verify that taking a limit of the expression of the higher-order beliefs, i.e., expression (22), in Proposition 2 at $\rho_k=0$ produces the same expressions of $\bar{E}_t\left[k_{t+1}\right]$ and $\bar{E}_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right]$ given in the text.

a lower weight relative to the typical Bayesian weight, i.e., $\frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}} \frac{\frac{1}{q}+\frac{1}{m}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}} < \frac{\frac{1}{q}}{\frac{1}{q}+\frac{1}{m}+\frac{1}{n}}$, while the prior is assigned a higher weight. As a result, $\bar{E}_t[k_{t+2}] \neq \bar{E}_t^2[k_{t+2}]$, consistent with literature on the role of public information in coordination settings (Morris and Shin 2002). Notice in a standard model without disagreements ($n = \infty$), each firm's information set contains only public information, no "overweighting" takes place, making the higher-order-beliefs degenerate. In the special case of $\rho_k = 0$, for beliefs higher than the second-order, the higher-order expectations become degenerate and equal to the prior $\bar{k}$, i.e., $\bar{E}_t^l[k_{t+l}] \equiv \bar{k}$ for $l > 2$. This is because period-t firms only receive private signals about, and thereby disagree on, the central bank's future inflation incentive up to period $t+2$. For any other future k_{t+l}, period-t firms share the same common prior $\bar{k}$ and agree with each other. Such perfect agreement among firms makes the higher-order beliefs that are higher than the second-order degenerate.

In the general case of serially correlated inflation incentive k_t ($\rho_k \neq 0$), the entire hierarchy of higher-order beliefs, including the ones that are higher than the second-order belief, remain nondegenerate. This is because, since k_t is serially correlated, period-t firms can utilize their private signals about k_{t+1} and k_{t+2} to forecast future $\{k_{t+l}\}_{l=3}^{\infty}$, thus disagreeing with each other in their beliefs about all of the central bank's future inflation incentive. Such disagreement in turn makes the higher-order beliefs $\left\{\bar{E}_t^l[k_{t+l}]\right\}_{l=3}^{\infty}$ nondegenerate. The proof of Proposition 2 contains the derivation of these expectations explicitly.

3.4 Symmetric Information Benchmark

Before we characterize the equilibrium in the model with informational imperfection, for comparison purposes, consider an identical model except no firm receives any signal about future inflation incentives (see Clarida, Gali and Gertler 1999, sections 3 and 4.1 for similar results.). It is well-known in this setting that under discretionary monetary policy, the equilibrium output and inflation contain an inflation bias driven by k_t and $\bar{k}$.

$$\begin{aligned}
\pi_t^* &= \frac{\alpha}{\alpha(1-\beta\rho_u)+\lambda^2}u_t + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right), \qquad (27)\\
x_t^* &= -\frac{\lambda}{\alpha(1-\beta\rho_u)+\lambda^2}u_t + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right),\\
i_t^* &= \frac{g_t}{\phi} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\rho_u+\lambda(1-\rho_u)/\phi}{\alpha(1-\beta\rho_u)+\lambda^2}u_t + \frac{\alpha\lambda\rho_k-\alpha(1-\beta\rho_k)(1-\rho_k)/\phi}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right).
\end{aligned}$$

The sources of aggregate output and inflation fluctuations are shocks to the Phillips curve and central bank's inflation incentive. Equilibrium nominal interest rate also reacts to shocks to the dynamic "IS" curve.

4 Equilibrium in Closed-Form

In this section, we first derive, in closed-form, the equilibrium inflation, output gap and nominal interest rate under the imperfect information environment for the special case of $j = 2$ (i.e., firms receive only two-period ahead k_{t+2}). Then, we conduct comparative stochastic dynamic analysis of how information imperfections affect the volatilities of equilibrium inflation and output.

4.1 The Stochastic Stationary Equilibrium

Substituting the expressions for the higher-order expectations $\bar{E}_t^l [k_{t+l}]$ given by equation (22) into the solution for π_t (in equation 21) and then x_t (in equation 16) gives the equilibrium inflation π_t^{**} and output gap x_t^{**}. The equilibrium nominal interest rate can be derived by substituting the pair (π_t^{**}, x_t^{**}) into the IS curve (7), giving the complete equilibrium characterization.

Proposition 3 *Assume $j = 2$, the equilibrium $\{\pi_t^{**}, x_t^{**}, i_t^{**}\}$ is given by:*

$$\begin{aligned}
\pi_t^{**} &= \frac{\alpha u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t - \bar{k}\right) \\
&\quad + \frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l[k_{t+l}]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right), \\
x_t^{**} &= -\frac{\lambda u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right) \\
&\quad - \frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l[k_{t+l}]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right), \\
i_t^{**} &= \frac{g_t}{\phi} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\rho_u+\lambda(1-\rho_u)/\phi}{\alpha(1-\beta\rho_u)+\lambda^2}u_t + \frac{\alpha\lambda\rho_k-\alpha(1-\beta\rho_k)(1-\rho_k)/\phi}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right) \\
&\quad + \frac{1}{\phi}\left[\frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l[k_{t+l}]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right)\right],
\end{aligned} \tag{28}$$

where the "demeaned" higher-order beliefs:

$$\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)=\left[1-w\left(l\right)\right]\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\nu_{t+1}+\eta_{t+1}\right)+w\left(l\right)\left(\frac{\nu_{t+2}+\rho_k\nu_{t+1}+\eta_{t+2}}{\rho_k}\right). \tag{29}$$

Proposition 3 shows that the equilibrium inflation π_t^{**} and hence the output gap x_t^{**} are determined by three factors, the contemporaneous (cost-push) shock u_t, the inflation bias $\frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k}+\frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\left(k_t-\bar{k}\right)$, and the higher-order expectations $\bar{E}_t^l\left[k_{t+l}\right]$. The first two factors have been extensively examined in the literature and appear even in the benchmark model without informational imperfection (see equation (27)). Specifically, consistent with standard results (Clarida, Gali and Gertler 1999), we verify that the cost-push shock u_t is inflationary. In addition, consistent with Barro and Gordon (1983), we find that the discretion in monetary policy can lead to a persistent inflation bias $\frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k}$.

In addition to the two well-known effects in the literature, the proposition above shows that the combination of the discretion in the monetary policy and the disagreement among firms can lead to another potentially detrimental effect, as captured in the third terms of the equilibrium inflation and output gap. Through the channel of the induced-coordination problem we identify, the discretionary monetary policy causes the equilibrium inflation and output to react to firms' higher-order beliefs about the central bank's inflation incentive, leading to heightened fluctuations in output and inflation. Our findings thereby suggest that the combination of lack of commitment by the central bank and the imperfect information known to firms makes the central bank less capable to stabilize output and inflation, as the central bank cannot resist the temptation to react to the noise contained in firms' imperfect information.

The source of the heightened output and inflation fluctuations come from the volatilities of the primitive variables in our model. Specifically, the equilibrium inflation and output will not only respond to the central bank's current inflation incentive k_t, but also to the noises, $\left\{\eta_{t+1},\eta_{t+2}\right\}$, contained in firms' average signals $\{\bar{s}_{t+1},\bar{s}_{t+2}\}$, as well as innovations in the central bank's future inflation incentive, $\{\nu_{t+1},\nu_{t+2}\}$. Furthermore, the coordination problem induced by the discretionary monetary policy makes the destabilizing effect of the monetary policy more prominent, due to a "multiplier" effect. When forecasting the forecasts of others, firms' average forecast is determined by a hierarchy of higher-order beliefs, each of which depends on the noises in firms' current information. As the monetary policy reacts to firms' forecast, the entire hierarchy of higher-order beliefs enters into the equilibrium inflation and output and the noise contained in these beliefs leads to heightened volatility. In particular, equation (28) shows precisely that since the same private signals $\left\{s_{t+1}^i, s_{t+2}^i\right\}$ are used for each level in the individual higher-order belief hierarchy, the common information noise $\left\{\eta_{t+1}, \eta_{t+2}\right\}$ in these private signals is retained at every term of the higher-order beliefs, magnifying the noises contained in the average inflation forecast. When the central bank responds to the average forecast, the magnified information noises enter into aggregate inflation and output, generating heightened macro fluctuations.

4.2 Comparative Stochastic Dynamic Analysis

In the face of the heightened volatility caused by firms' imperfect information, the conventional wisdom would suggest that reducing the volatilities of the two informational shocks, η_{t+j} (average forecast-error) and ε_{t+j}^i (degree of disagreement) would be desirable. We find that this intuition does not hold generally. Importantly, we show that reducing the volatilities of the two informational shocks can increase the macro fluctuations by inducing the monetary policy to respond more aggressively to firms' imperfect information and the associated noises. To see the effect of informational properties on volatilities, from Proposition 3, the volatility of inflation is computed as:

$$\begin{aligned} Var\left(\pi_t^{**}\right) \quad = \quad & \left(\frac{\alpha}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}\right)^2 \frac{\sigma_u^2}{1-\rho_u^2} + \left(\frac{\alpha\lambda}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\right)^2 Var\left(k_t\right) \\ & + \left(\frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\right)^2 Var\left(\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)\right)\right), \end{aligned} \tag{30}$$

where the first term represents the volatility stemming from the shock u_t, the second term represents the volatility stemming from the central bank's *current* inflation incentive k_t, and the third term represents the volatility stemming from higher-order expectations about the central bank's *future* inflation incentive. By the first-order condition, $x_t^{**} = -\frac{\lambda}{\alpha}\pi_t^{**} + k_t$, the volatility of x_t^{**} is proportional to the volatility of π_t^{**} and the two share similar properties. Thus we will focus on analyzing the volatility of π_t^{**}. For notational convenience, we define the sensitivities of the equilibrium inflation to the future signals $\bar{s}_{t+1}$ and $\bar{s}_{t+2}$ as

$$\begin{aligned} W_{\bar{s}_{t+1}}(m,n) &= \frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left\{[1-w(l)]\frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\right\}, \\ W_{\bar{s}_{t+2}}(m,n) &= \frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\frac{w(l)}{\rho_k}, \end{aligned} \tag{31}$$

which depend on the informational properties, the average forecast error m and the degree of disagreement n. Using notations $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$, $Var(\pi_t^{**})$ can be rewritten as:

$$\begin{aligned} &\left(\frac{\alpha}{\alpha(1-\beta\rho_u)+\lambda^2}\right)^2\frac{\sigma_u^2}{1-\rho_u^2}+\left(\frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\right)^2 Var(k_t) \\ &+\left[W_{\bar{s}_{t+1}}(m,n)+\rho_k W_{\bar{s}_{t+2}}(m,n)\right]^2 Var(\nu_{t+1})+\left[W_{\bar{s}_{t+1}}(m,n)\right]^2 Var\left(\eta_{t+1}\right) \\ &+\left[W_{s_{t+2}}(m,n)\right]^2\left[Var(\nu_{t+2})+Var\left(\eta_{t+2}\right)\right]. \end{aligned} \tag{32}$$

Because $Var(\nu_{t+1}) = Var(\nu_{t+2}) = \frac{1}{q}$ and $Var\left(\eta_{t+1}\right) = Var\left(\eta_{t+2}\right) = \frac{1}{m}$. Therefore, $Var(\pi_t^{**})$ becomes

$$\begin{aligned} &\left(\frac{\alpha}{\alpha(1-\beta\rho_u)+\lambda^2}\right)^2\frac{\sigma_u^2}{1-\rho_u^2}+\left(\frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\right)^2 Var(k_t) \\ &+\frac{\left[W_{\bar{s}_{t+1}}(m,n)+\rho_k W_{\bar{s}_{t+2}}(m,n)\right]^2+\left[W_{s_{t+2}}(m,n)\right]^2}{q}+\frac{\left[W_{\bar{s}_{t+1}}(m,n)\right]^2+\left[W_{s_{t+2}}(m,n)\right]^2}{m}. \end{aligned} \tag{33}$$

Equation (33) suggests that in addition to the volatility driven by the shocks u_t and k_t, inflation volatility is also driven by two other shocks. The third term in (33) represents the fundamental volatility stemming from the innovations in the central bank's future inflation incentive $\{\nu_{t+1}, \nu_{t+2}\}$ and the fourth term is the non-fundamental volatility stemming from the noises in firms' signals, i.e., $\left\{\eta_{t+1}, \eta_{t+2}\right\}$. Equation (33) shows that the informational properties can influence the macro fluctuations in two ways. First, improving the precision of the average forecast-error (increasing m) directly reduces the size of the noises $\left\{\eta_{t+1}, \eta_{t+2}\right\}$ in the equilibrium monetary policy, leading to less volatility in the output and inflation. We capture this effect in the $\frac{1}{m}$ term in (33) and call this effect a *noise-diminishing* effect. Second, when either the precision of average forecast-error or the agreement among firms increases, average forecast becomes more sensitive to the firms' imperfect information, thus the average forecast error. The central bank, in turn, is tempted to exploit the higher sensitivity by reacting more aggressively to the average forecast including its error; unfortunately this reaction adds more volatility to equilibrium inflation and output. In other words, increasing m or n can increase the sensitivity of the monetary policy to firms' signals and noises (i.e., $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$). We capture this effect in $W_{\bar{s}_{t+1}}(m,n)$ and $W_{\bar{s}_{t+2}}(m,n)$ and call this effect a *sensitivity* effect. Whether improving firms' information (increasing m and n) reduces the volatilities thereby depends on the trade-off between the sensitivity effect and the noise-diminishing effect. We summarize the effect of the informational properties on the volatilities of inflation and output in the proposition below.

Proposition 4 *Information properties* (m,n) *influence the volatilities of inflation and output as follows:*

1. *Volatilities are strictly increasing in* n*, i.e., more agreement always increases volatilities;*

2. *There exists a unique* $\hat{n}$*, such that volatilities strictly decrease in* m *if and only if* $n < \hat{n}$*, i.e., more accurate average forecast decreases volatilities when disagreement is sufficiently high.*

Proposition 4 suggests that holding fixed the average forecast-error volatility (m), a higher degree of agreement among agents (increase n) leads to higher fluctuations in output and inflation. On the other hand, holding the degree of disagreement fixed, reducing the size of the average forecast-error has a non-monotonic effect on the volatility. We find that increasing m helps to stabilize inflation and output if and only if the disagreement among the firms is sufficiently high.

The intuition for these results is due to a trade-off between the *noise-diminishing* and the *sensitivity* effects. Specifically, as we explained earlier, the lack of commitment by the central bank induces an implicit coordination motive among the firms, making it necessary for an individual firm to forecast the forecasts of others. That is, in forming its best forecast of the future inflation, a firm uses its information not only to estimate the central bank's inflation incentive but also others' beliefs about the incentive. We call the first use of information as the *fundamental* value of information and the second use as the *strategic* value of information. Under the information structure specified in our model, improvements in the precision of the average forecast m and the agreement n play different roles in affecting the two uses of information (see Liang and Zhang 2019). First, increasing either m or n diminishes the size of (common or idiosyncratic) noises and moves the firms' signals closer to the central bank's true target, which enhances the fundamental value of information. Second, increasing n increases the strategic value of information while increasing m decreases the strategic value of information. This is because the strategic value of information is determined by the correlation between firms' private signals, $corr\left(s^i_\tau, s^{i'}_\tau\right) = \frac{\frac{1}{m}+\frac{1}{q}}{\frac{1}{m}+\frac{1}{q}+\frac{1}{n}}$ for $i \neq i'$, which is strictly increasing in n but decreasing in m. Intuitively, increasing m reduces the size of common noises and hence the common variation among the firms' signals, reducing the correlation between the signals while increasing n decreases the size of idiosyncratic noises and hence the idiosyncratic variation, increasing the correlation.

The different role of m and n in influencing the value of information determines their effects on the volatilities. We first explain the effect of higher agreement. Since increasing n (higher agreement) increases both the fundamental and the strategic value of the information, all firms respond more sensitively to their signals $\left\{s^i_{t+1}, s^i_{t+2}\right\}$ in forming their forecasts ($W_{\bar{s}_{t+1}}(m, n)$ and $W_{\bar{s}_{t+2}}(m, n)$ both increase). After the idiosyncratic noises $\left\{\varepsilon^i_{t+1}, \varepsilon^i_{t+2}\right\}$ are diversified away in the aggregation, the average expectation of the firms becomes more responsive to the average signals $\{\bar{s}_{t+1}, \bar{s}_{t+2}\}$. This is the sensitivity effect of increasing n. When the central bank cannot commit, it is tempted to respond more to the aggregate expectation, making its monetary policy more sensitive to the errors in firms' average expectation as well. As a result, the equilibrium inflation induced by the monetary policy is driven by the errors in the aggregate expectation to a larger extent and becomes more volatile.

The effect of increasing m differs from that of increasing n in two ways. First, increasing m increases the fundamental value of the information but decreases the strategic value. Overall, increasing m still increases the firms' sensitivity to their signals (increases $W_{\bar{s}_{t+1}}(m, n)$ and $W_{\bar{s}_{t+2}}(m, n)$). The higher sensitivity leads to higher volatilities, through the transmission mechanism illustrated above; however, this sensitivity effect of m is weaker than that of n because the decrease in the strategic value of the information led by higher m dampens the increase in the sensitivity. Second, in aggregating the firms' forecasts, the common noise $\left\{\eta_{t+1}, \eta_{t+2}\right\}$ is not diversified away as the idiosyncratic noises $\left\{\varepsilon^i_{t+1}, \varepsilon^i_{t+2}\right\}$. This captures the noise-diminishing effect of increasing m: a higher m directly diminishes the size of the common noise and makes the average expectation and hence the inflation less volatile. The net effect of m on the volatilities thus depends on the trade-off between the sensitivity effect and the noise-diminishing effect. When the disagreement is sufficiently high, the strategic value of information in forecasting the forecasts of others becomes important. Due to the adverse effect of m on the strategic value, the firms are more reluctant to respond to their information, despite that the increase in m improves the fundamental value. As a result, the sensitivity effect becomes weak and dominated by the noise-diminishing effect. Accordingly, increasing m leads to lower volatilities. Otherwise, when the disagreement is low, the strategic value of information becomes less important, making the sensitivity effect strong and dominate the noise-diminishing effect. In these cases, increasing m amplifies the volatilities.

5 Conclusion

With its simplicity, our paper makes a core argument for the inclusion of information diversity among agents in monetary policy discussions. A direct implication of our model is on the explanation and characterization of the observed inflation dynamics. Our model would suggest that the precision of the aggregate estimation of future inflation is a determinant of current inflation. In this regard, our paper is related to the voluminous macro-literature on inflation trend (Goodfriend and King 2012 and Ascari and Sbordone 2014). In these studies, firms are more sophisticated in their understanding of the inflation trend and adjust their pricing behavior (such as indexing). In an extension, we verify that our main qualitative results survive in a more general model (e.g., Woodford 2008) in which an inflation trend term is inserted into the New Keynesian Phillips curve.

More broadly, we view our paper as an attempt at constructing a positive understanding of the macro-economy under the information imperfections about the incentives of an authority player. Our paper is not directly concerned about how these information imperfections emerge endogenously from the *information production* of each player in the model, but any such studies should take into account the results of our paper. Recent interests in studying the communication strategies of the central bank are evidence of its perceived importance (see, e.g., Rudebusch and Williams 2006).

Aside from the information flow from the central bank to the marketplace, a more organic environment would also feature active private information activities. As shown in the Fed-watch literature, individual agents are motivated to acquire relevant information in anticipated information management by the central bank.

Finally, our paper raises issues that future studies could blend with other important considerations related to information and coordination. They include other coordination problems in macroeconomics (Cooper and John 1988; Baxter and King 1991; Kiyotaki and Wright 1989, etc.), robust policies by Hansen and Sargent (2007), and information role of financial market (King 1982; Baxter and King 1991).

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Appendix I: Proofs

Proof of Proposition 1

Proof. This can be verified by iterating (17). ■

Proof of Proposition 2

Proof. For our convenience, we define the vectors of firm i's demeaned signals at period t and the vector of the demeaned average signals at period t as:

$$S_t^i = \begin{bmatrix} k_t - \bar{k} \\ s_{t+1}^i - \bar{k} \\ s_{t+2}^i - \bar{k} \end{bmatrix}, \bar{S}_t = \begin{bmatrix} k_t - \bar{k} \\ \bar{s}_{t+1} - \bar{k} \\ \bar{s}_{t+2} - \bar{k} \end{bmatrix}. \tag{34}$$

The variance of S_t^i as

$$Var\left(S_t^i\right) = \Sigma = \begin{bmatrix} \frac{1}{q_k} & \frac{\rho_k}{q_k} & \frac{\rho_k^2}{q_k} \\ \frac{\rho_k}{q_k} & \frac{1}{q_k} + \frac{1}{m} + \frac{1}{n} & \frac{\rho_k}{q_k} \\ \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} + \frac{1}{m} + \frac{1}{n} \end{bmatrix}, \tag{35}$$

and the covariance between $\bar{S}_{t+1}$ and S_t^i as

$$Cov\left(\bar{S}_{t+1}, S_t^i\right) = \Omega = \begin{bmatrix} \frac{\rho_k}{q_k} & \frac{1}{q_k} & \frac{\rho_k}{q_k} \\ \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} + \frac{1}{m} \\ \frac{\rho_k^3}{q_k} & \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} \end{bmatrix}. \tag{36}$$

In particular, we define the first row of Ω, the covariance between k_{t+1} and S_t^i, as

$$\Omega^{\text{row 1}} = L = \begin{bmatrix} \frac{\rho_k}{q_k} & \frac{1}{q_k} & \frac{\rho_k}{q_k} \end{bmatrix}. \tag{37}$$

We now derive the hierarchy of higher-order beliefs. In the first-order belief in period t, each firm's forecast of k_{t+1} is

$$E_t^i\left[k_{t+1}\right] = \bar{k} + L\Sigma^{-1}S_t^i, \tag{38}$$

and the average forecast is

$$\bar{E}_t\left[k_{t+1}\right] = \bar{k} + L\Sigma^{-1}\bar{S}_t. \tag{39}$$

Building on the first-order expectation, now move to the the second-order belief. For firm-i, its period-t belief about the aggregate period $t+1$ belief about the central banks period $t+2$ incentive becomes

$$E_t^i\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right] = \bar{k} + E_t^i\left[L\Sigma^{-1}\bar{S}_{t+1}\right] = \bar{k} + L\Sigma^{-1}E_t^i\left[\bar{S}_{t+1}\right], \tag{40}$$

where $E_t^i\left[\bar{S}_{t+1}\right] = \Omega\Sigma^{-1}S_t^i$. Therefore, the average second-order belief becomes:

$$\bar{E}_t\left[\bar{E}_{t+1}\left[k_{t+2}\right]\right] = \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}\bar{S}_t. \tag{41}$$

Notice that the law of iterated expectation fails, i.e., $\bar{E}_t^2\left[k_{t+2}\right] \neq \bar{E}_t\left[k_{t+2}\right] = \bar{k} + \begin{bmatrix} \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} \end{bmatrix} \Sigma^{-1}\bar{S}_t$, since $L\Sigma^{-1}\Omega = \begin{bmatrix} \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k}\frac{\left(\frac{1}{m}+\frac{1}{q}\right)\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)+\left(\frac{1}{mq}+\frac{1}{mn}+\frac{2}{nq}+\frac{1}{n^2}\right)\rho_k^2}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2} \end{bmatrix} \neq \begin{bmatrix} \frac{\rho_k^2}{q_k} & \frac{\rho_k}{q_k} & \frac{1}{q_k} \end{bmatrix}$ for $n \neq \infty$ (i.e., there is some disagreement among firms). In particular, we verify that in $\bar{E}_t^2\left[k_{t+2}\right]$, the signal $\bar{s}_{t+2}$ is weighted less than in $\bar{E}_t\left[k_{t+2}\right]$. Moreover, for the third-order belief, firm-i's period-t belief about the aggregate period $t+1$ belief about the aggregate period $t+2$ belief about the central banks period $t+3$ incentive becomes

$$E_t^i\left[\bar{E}_{t+1}\left[\bar{E}_{t+2}\left[k_{t+3}\right]\right]\right] = \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}E_t^i\left[\bar{S}_{t+1}\right] = \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}\Omega\Sigma^{-1}S_t^i, \tag{42}$$

and thus the average third-order belief becomes:

$$\bar{E}_t\left[\bar{E}_{t+1}\left[\bar{E}_{t+2}\left[k_{t+3}\right]\right]\right] = \bar{k} + L\Sigma^{-1}\Omega\Sigma^{-1}\Omega\Sigma^{-1}\bar{S}_t = \bar{k} + L\Sigma^{-1}\left(\Omega\Sigma^{-1}\right)^2\bar{S}_t. \tag{43}$$

Keeping iterating $\bar{E}_t^3\left[k_{t+3}\right]$ characterizes the entire hierarchy of higher-order beliefs with

$$\bar{E}_t^l\left[k_{t+l}\right] = \bar{k} + L\left(\Sigma^{-1}\Omega\right)^{l-1}\Sigma^{-1}\bar{S}_t. \tag{44}$$

To derive $\bar{E}_t^l\left[k_{t+l}\right]$, we make an eigenvalue decomposition on $\Sigma^{-1}\Omega$, such that,

$$\Sigma^{-1}\Omega = Q\Lambda Q^{-1}, \tag{45}$$

where Λ is a diagonal matrix with the eigenvalue of $\Sigma^{-1}\Omega$ on its diagonal, i.e.,

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\frac{1}{n}\frac{\rho_k}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2} & 0 \\ 0 & 0 & \rho_k \end{bmatrix}, \tag{46}$$

and Q is the associated matrix of eigenvectors. As a result,

$$\begin{aligned} \bar{E}_t^l\left[k_{t+l}\right] &= \bar{k} + LQ\Lambda^{l-1}Q^{-1}\Sigma^{-1}\bar{S}_t \qquad (47) \\ &= \bar{k} + LQ \begin{bmatrix} 0 & 0 & 0 \\ 0 & \left[\frac{\frac{1}{n}\frac{\rho_k}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}\right]^{l-1} & 0 \\ 0 & 0 & \rho_k^{l-1} \end{bmatrix} Q^{-1}\Sigma^{-1}\bar{S}_t, \end{aligned}$$

and can be simplified into

$$\bar{E}_t^l\left[k_{t+l}\right] = \bar{k} + \rho_k^{l-1}\left\{\left[1-w\left(l\right)\right]\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right] + w\left(l\right)\frac{\bar{s}_{t+2}-\bar{k}}{\rho_k}\right\}, \tag{48}$$

where $\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right] = \frac{q}{q+\frac{mn}{m+n}}\rho_k\left(k_t-\bar{k}\right) + \frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\bar{s}_{t+1}-\bar{k}\right)$ and

$$\begin{aligned} w\left(l\right) &= \frac{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2} \qquad (49) \\ &+\frac{\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)\frac{1}{q}\left[\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)\left(\frac{1}{m}+\frac{1}{q}\right)\right]\left\{1-\left[\frac{\frac{1}{n}\frac{1}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}\right]^{l-1}\right\}}{\left[\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2\right]^2\left\{1-\left[\frac{\frac{1}{n}\frac{1}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2}\right]\right\}}. \end{aligned}$$

Notice that since $\frac{\frac{1}{n}\frac{1}{q}}{\left(\frac{1}{m}+\frac{1}{n}\right)\frac{\rho_k^2}{q}+\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{q}\right)^2} < 1$, then $w\left(l\right)$ is strictly increasing in l. ■

Proof of Proposition 3

Proof. Substituting the expressions for the higher-order-beliefs terms into the expression for the inflation specified in Proposition 1, we have

$$\begin{aligned}\pi_t^{**} &= \frac{\alpha u_t}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}+\frac{\alpha\lambda}{\alpha\left(1-\beta\right)+\lambda^2}\bar{k}+\frac{\alpha\lambda}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\left(k_t-\bar{k}\right) \\ &\quad +\frac{\alpha\lambda}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)\right),\end{aligned} \tag{50}$$

where the "demeaned" higher-order beliefs is:

$$\begin{aligned}&\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right) \\ =\ &\left[1-w\left(l\right)\right]\left[\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right]-\rho_k\left(k_t-\bar{k}\right)\right]+w\left(l\right)\left(\frac{\bar{s}_{t+2}-\bar{k}}{\rho_k}-\rho_k\left(k_t-\bar{k}\right)\right).\end{aligned} \tag{51}$$

with

$$\begin{aligned}\bar{E}_t\left[k_{t+1}-\bar{k}|s_{t+1}^i,k_t\right]-\rho_k\left(k_t-\bar{k}\right) &= \frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\bar{s}_{t+1}-\bar{k}-\rho_k\left(k_t-\bar{k}\right)\right) \\ &= \frac{\frac{mn}{m+n}}{q+\frac{mn}{m+n}}\left(\eta_{t+1}+\nu_{t+1}\right),\end{aligned} \tag{52}$$

and

$$\frac{\bar{s}_{t+2}-\bar{k}}{\rho_k}-\rho_k\left(k_t-\bar{k}\right)=\frac{\nu_{t+2}+\rho_k\nu_{t+1}+\eta_{t+2}}{\rho_k}. \tag{53}$$

By the first-order condition, the equilibrium output is

$$\begin{aligned}x_t^{**} &= -\frac{\lambda}{\alpha}\pi_t^{**}+k_t \\ &= -\frac{\lambda u_t}{\alpha\left(1-\beta\rho_u\right)+\lambda^2}+\frac{\alpha\left(1-\beta\right)}{\alpha\left(1-\beta\right)+\lambda^2}\bar{k}+\frac{\alpha\left(1-\beta\rho_k\right)}{\alpha\left(1-\beta\rho_k\right)+\lambda^2}\left(k_t-\bar{k}\right) \\ &\quad -\frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}}-\rho_k\left(k_t-\bar{k}\right)\right).\end{aligned} \tag{54}$$

The equilibrium nominal interest rate can be derived by substituting the pair (π_t^{**}, x_t^{**}) into the IS curve (7):

$$i_t^{**}=\frac{E_t^H x_{t+1}^{**}}{\phi}+\frac{g_t}{\phi}-\frac{x_t^{**}}{\phi}+E_t^H\pi_{t+1}^{**}, \tag{55}$$

where given the information set of the household, $I_t^H = \left\{ \{u_\tau\}_{\tau=0}^t, \{g_\tau\}_{\tau=0}^t, \{k_\tau\}_{\tau=0}^t \right\}$,

$$\begin{aligned} E_t^H x_{t+1}^{**} &= -\frac{\lambda \rho_u u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha(1-\beta\rho_k)}{\alpha(1-\beta\rho_k)+\lambda^2}\rho_k\left(k_t-\bar{k}\right), \\ E_t^H \pi_{t+1}^{**} &= \frac{\alpha\rho_u u_t}{\alpha(1-\beta\rho_u)+\lambda^2} + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} + \frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\rho_k\left(k_t-\bar{k}\right), \end{aligned} \tag{56}$$

as a result,

$$\begin{aligned} i_t^{**} &= \frac{g_t}{\phi} + \frac{u_t}{\alpha(1-\beta\rho_u)+\lambda^2}\left[\alpha\rho_u + \frac{\lambda(1-\rho_u)}{\phi}\right] + \frac{\alpha\lambda}{\alpha(1-\beta)+\lambda^2}\bar{k} \\ &+ \frac{\alpha\left(k_t-\bar{k}\right)}{\alpha(1-\beta\rho_k)+\lambda^2}\left[\lambda\rho_k - \frac{(1-\beta\rho_k)(1-\rho_k)}{\phi}\right] \\ &+ \frac{1}{\phi}\left[\frac{\lambda^2}{\alpha+\lambda^2}\frac{\alpha\beta}{\alpha+\lambda^2}\sum_{l=1}^{\infty}\left(\frac{\alpha\beta\rho_k}{\alpha+\lambda^2}\right)^{l-1}\left(\frac{\bar{E}_t^l\left[k_{t+l}\right]-\bar{k}}{\rho_k^{l-1}} - \rho_k\left(k_t-\bar{k}\right)\right)\right]. \end{aligned} \tag{57}$$

■

Proof of Proposition 4

Proof. Notice that $x_t^{**} = -\frac{\lambda}{\alpha}\pi_t^{**} + k_t$. Thus

$$Var_t\left(x_t^{**}\right) = \frac{\lambda^2}{\alpha^2}Var_t\left(\pi_t^{**}\right) + Var_t\left(k_t\right) - \frac{2\lambda}{\alpha}Cov_t\left(\pi_t^{**}, k_t\right), \tag{58}$$

where $Var_t(k_t) = \frac{1}{q(1-\rho_k)^2}$ and $Cov_t\left(\pi_t^{**}, k_t\right) = \frac{\alpha\lambda}{\alpha(1-\beta\rho_k)+\lambda^2}\frac{1}{q(1-\rho_k)^2}$ are both independent of m and n. Therefore, the effects of (m,n) on $Var_t\left(x_t^{**}\right)$ are the same as their effects on $Var_t\left(\pi_t^{**}\right)$.

One can verify $\frac{\partial Var_t(\pi_t^{**})}{\partial n} < 0$ by directly computing the derivatives. For the sign of $\frac{\partial Var_t(\pi_t^{**})}{\partial m}$, first, one can verify that at $n=0$, $\frac{\partial Var_t(\pi_t^{**})}{\partial m} = 0$, $\frac{\partial}{\partial n}\left(\frac{\partial Var_t(\pi_t^{**})}{\partial m}\right) = 0$ and $\frac{\partial^2}{\partial n^2}\left(\frac{\partial Var_t(\pi_t^{**})}{\partial m}\right) = -\frac{2\left(1+\rho_k^2\right)}{m^2q^2\left(1-\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\right)^2} < 0$. As a result, for n close to 0, $\lim_{n\to 0^+}\frac{\partial Var_t(\pi_t^{**})}{\partial m} < 0$. Second, at $n=\infty$,

$$\begin{aligned} &\frac{\partial Var_t\left(\pi_t^{**}\right)}{\partial m} \\ &= \frac{\left(1+\left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^2\right)(m+q)^2 + 2\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\left(q^2-m^2\right) + \left(\left(\frac{\alpha\beta}{\alpha+\lambda^2}m\right)^2+q^2\right)\rho_k^2}{\left(1-\frac{\alpha\beta}{\alpha+\lambda^2}\rho_k\right)^2\left[(m+q)^2+mq\rho_k^2\right]^2} > 0. \end{aligned} \tag{59}$$

Therefore, by the intermediate value theorem, there exists a $\hat{n} > 0$, such that $\frac{\partial Var_t(\pi_t^{**})}{\partial m} = 0$.

Lastly, we verify that such a $\hat{n}$ is also unique. More specifically, we verify that $\frac{\partial Var_t(\pi_t^{**})}{\partial m} = 0$ can be reduced into $P(n) = 0$ and $P(n)$ is a fourth-order polynomial of n,

$$P(n) = \kappa_1 n^4 + \kappa_2 n^3 + \kappa_3 n^2 + \kappa_4 n^3 + \kappa_5, \tag{60}$$

where the expressions of the coefficients $\{\kappa_i\}_{i=1}^5$ are available upon requests. We verify that $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_5 < 0$ and the signs of κ_3 and κ_4 are ambiguous. However, it is impossible to have $\kappa_3 < 0$ and $\kappa_4 > 0$ at the same time. As a result, there can be the following three possible scenarios of the signs of $\{\kappa_i\}_{i=1}^5$:

κ_1	κ_2	κ_3	κ_4	κ_5
$+$	$+$	$+$	$+$	$-$
$+$	$+$	$+$	$-$	$-$
$+$	$+$	$-$	$-$	$-$

,

where "+" means positive and "$-$" means negative. Notice that for the polynomial $P(n)$, there is one sign change in its coefficients. Therefore, by the Descartes' rule of signs, the polynomial $p(n)$ has a unique positive root. That is, there exists a unique $\hat{n}$ that makes $\frac{\partial Var_t(\pi_t^{**})}{\partial m} = 0$. As a result, $\frac{\partial Var_t(\pi_t^{**})}{\partial m} < 0$ if and only if $n < \hat{n}$. ■